ttrr Documentation

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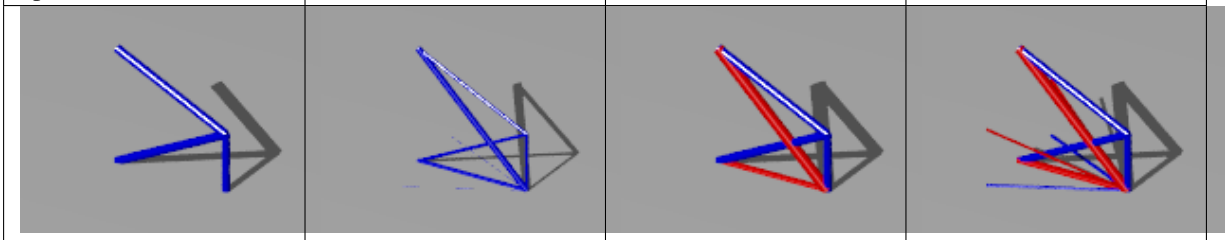
CONTENTS

optimal	robust	redundant	robust + redundant
			
			see *Example 3*

ttrr is an acronym for Truss Topology Robust Redundant.

ttrr is a software package to do topology optimization of a truss with respect to robustness and redundancy.

This documentation is available as:

- html http://ttrr.cadae.de/ (incl. downloads)
- pdf: `ttrr.pdf`
- epub: `ttrr.epub`

Contents:

CHAPTER

ONE

DOWNLOAD

The available versions are:

- `ttrr-latest.tar.gz` (alpha)
  md5sum:
- `ttrr-2012-06-26.tar.gz` (alpha)
  md5sum: f4323db1c107379a664c65fcdfa265fb
- `ttrr-2012-05-05.tar.gz` (alpha)
  md5sum: 421f4e94be1cb74f5068224cf83f8af2
- `ttrr-2012-03-16.tar.gz` (alpha)
  md5sum: cd26fb4d145d24f87b4b061993e08162
- `ttrr-2012-03-10.tar.gz` (alpha)
  md5sum: 579975a0f99ce6b3efb6d69f95d03c7c
- ttrr-2012-03-09.tar.gz (alpha)
  md5sum: ed4528c399c0e6bef617c4045ab00b53
- ttrr-2012-02-23.tar.gz (alpha)
  md5sum: 6f5a4f57ba761b923ea6828aeee53057
- ttrr-2012-02-22.tar.gz (pre-alpha)
  md5sum: a007c94ae1e72236d49b6b30d16e266a

CHAPTER

TWO

TTRR – TRUSSTOPOLOGYROBUS-TREDUNDANT

platform Unix

synopsis ttrr is a software package to do topology optimization of a truss with respect to robustness and redundancy.

TrussTopologyRobustRedundant

Contents

2.1 description

ttrr is a software package to do topology optimization of a truss with respect to robustness and redundancy — see literatur.

There are 2 parts of ttrr:

- **library:** You can use `ttrr` as a library to import to your own python-code. To calculate trusses `ttrr` needs **ttrr_calculate** resp. ttrr_calculate.cc
- **command line tool:** you can use `ttrr_tools.py` as a command line tool. for detail:

```
ttrr_tools.py -h
```

functions to include from ttrr resp. ttrr.py: (in alphabetical order)

- `compute_disturbed_forces()`
- `nodenumberfromcoordinates()`
- `barsnodesongrid2d()`
- `barsnodesongrid3d()`
- `ttrr()`
- `ttrr_redundant()`
- `ttrr_robust()`

2.2 typical parameters

dim [integer] dimension 2 or 3 of the design space

nodes [numpy.array] is a matrix to represent all nodal points with:

- nodes(k,:) are the nodal points.
- nodes(:,1) are the x-coordinates of the nodal points.
- nodes(:,2) are the y-coordinates of the nodal points.
- nodes(:,3) are the z-coordinates of the nodal points. (mostly optional)

bars [numpy.array] is a vector to represent all trusses:

- bars(s,:) are numbers of the trusses.
- bars(:,1) is the number of the first nodal point.
- bars(:,2) is the number of the second nodal point.

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed.

loads [scipy.sparse.lil_matrix] are the load forces.

loads(:,k) are the forces for the k-th load case

smax [float] is the maximum allowed cross section of a truss

opt [integer] choose the optimization routine: (optional)

- 0 glp_simplex (default, iff redundancy == 0)
- 1 glp_interior (only for redundancy == 0)
- 2-5 glp_mip (for redundancy != 0)
  - 2 do not search greedy-solution (default, for redundancy != 0)
  - 3 search greedy-solution and go on
  - 4 search greedy-solution, write greedy solution and go on
  - 5 search greedy-solution and stop

2.3 functions

ttrr.**compute_disturbed_forces**(*loads*, *dim*, *p*, *art=0*)

forces_disturbed = compute_disturbed_forces(loads,dim,p,art=0)

calculates perturbed forces from loads and returns them forces_disturbed. For nl load cases in loads you get in forces_disturbed (2^dim)*nl load cases.

Parameters:

dim [integer] dimension 2 or 3 of the design space

p [float] percentage of magnitude of the unperturbed force is used for the perturbation. p*norm(loads[:,k]) is the magnitude of the perturbation of the load case k.

art [integer] chooses the kind of the perturbation: (optional)

art == 0 generates perturbations in every direction of the original forces and scales them. The maximal magnitude of the forces is the same. (default)

art == 1 generates perturbations in every direction of the original forces. The maximal magnitude of the forces changes.

art [integer] waehlt die Art die gestoerrten Kraefte zu erstellen (optional)

art == 0 erzeuge Stoerrung um p*norm(kraft) in jede Richtung. skaliere die Kraefte so, dass die maximale Kraftgroesse gleich bleibt (default)

art == 1 erzeuge Stoerrung um p*norm(kraft) in jede Richtung. Achtung: maximale Kraftgroesse aendert sich

Author: Daniel Mohr.

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`ttrr.`**`nodenumberfromcoordinates`**(*nodes*, *x*, *y*, *z=0*, *eps=1.4901161193847656e-08*)

kk = nodenumberfromcoordinates(nodes,x,y,z=0,eps=_epsilon) calculates the number of a node from the coordinates.

nodes is a matrix to represent all nodal points with nodes(k,:) are the nodal points nodes(:,1) are the x-coordinates of the nodal points nodes(:,2) are the y-coordinates of the nodal points nodes(:,3) are the z-coordinates of the nodal points (optional)

x,y,z gives the coordinate of the wanted node. If the dimension of the design space (it is get from the dimension of nodes) is 2 z is ignored. z is optional.

eps is a bound for the possible difference between the coordinate of the wanted node and given coordinate.

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`ttrr.`**`barsnodesongrid2d`**(*gx0*, *gx1*, *gy0*, *gy1*, *nelx*, *nely*, *mounting=0*)

[nodes,bars]=barsnodesongrid2d(gx0,gx1,gy0,gy1,nelx,nely,mounting=0)

creates all bars and nodes which make sense. gx0, gx1, gy0 and gy1 describes the edges (gx0,gy0), (gx0,gy1), (gx1,gy0) and (gx1,gy1) of the rectangular design space. In this rectangle it will be nelx*nely nodes calculated. With the assumption of an equidistant grid all meaningful bars are calculated. Bars which can be represent of others are ignored. For example a bar from (0,0) to (0,1) is ignored, if bars from (0,0) to (0,0.5) and from (0,0.5) to (0,1) exist.

Parameters:

gx0 [float] lower bound of the x-cooredinates to describe the rectangle

gx1 [float] upper bound of the x-cooredinates to describe the rectangle

gy0 [float] lower bound of the y-cooredinates to describe the rectangle

gy0 [float] upper bound of the y-cooredinates to describe the rectangle

nelx [integer] is the number of nodes in x-direction

nely [integer] is the number of nodes in y-direction

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed. (optional) If denoted bars fixed on both ends will be ignored.

Returns:

nodes [numpy.array] is a matrix to represent all nodal points with:

- nodes(k,:) are the nodal points.
- nodes(:,1) are the x-coordinates of the nodal points.
- nodes(:,2) are the y-coordinates of the nodal points.
- nodes(:,3) are the z-coordinates of the nodal points. (mostly optional)

bars [numpy.array] is a vector to represent all trusses:

- bars(s,:) are numbers of the trusses.
- bars(:,1) is the number of the first nodal point.
- bars(:,2) is the number of the second nodal point.

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`ttrr.`**`barsnodesongrid3d`**(*gx0*, *gx1*, *gy0*, *gy1*, *gz0*, *gz1*, *nelx*, *nely*, *nelz*, *mounting=0*, *areawithoutnode=0*, *mountingarea=0*, *maxstablaenge=-1*)

[nodes,bars]=barsnodesongrid3d(gx0,gx1,gy0,gy1,gz0,gz1,nelx, nely,nelz,mounting=0,areawithoutnode=0,maxstablaenge=-1) [nodes,bars,mounting]=barsnodesongrid3d(gx0,gx1,gy0,gy1,gz0,gz1,nelx,

nely,nelz,areawithoutnode=0,mountingarea=0, maxstablaenge=-1)

creates all bars and nodes which make sense. If mountingarea is denoted the fixed nodes are also calculated and returned. gx0, gx1, gy0, gy1, gz0 and gz1 describes the edges (gx0,gy0,gz0), (gx0,gy0,gz1), (gx0,gy1,gz0), (gx0,gy1,gz1) (gx1,gy0,gz0), (gx1,gy0,gz1), (gx1,gy1,gz0) and (gx1,gy1,gz1) of the cuboid-shaped design space. In this cuboid it will be nelx*nely*nelz nodes calculated. With the assumption of an equidistant grid all meaningful bars are calculated. Bars which can be represent of others are ignored. For example a bar from (0,0,0) to (0,0,2) is ignored, if bars from (0,0,0) to (0,0,1) and from (0,0,1) to (0,0,2) exist.

Parameters:

gx0 [float] lower bound of the x-cooredinates to describe the rectangle

gx1 [float] upper bound of the x-cooredinates to describe the rectangle

gy0 [float] lower bound of the y-cooredinates to describe the rectangle

gy0 [float] upper bound of the y-cooredinates to describe the rectangle

gz0 [float] lower bound of the z-cooredinates to describe the rectangle

gz0 [float] upper bound of the z-cooredinates to describe the rectangle

nelx [integer] is the number of nodes in x-direction

nely [integer] is the number of nodes in y-direction

nelz [integer] is the number of nodes in z-direction

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed. (optional) If denoted bars fixed on both ends will be ignored.

areawithoutnode [numpy.array] is a matrix describing areas without nodes (optional) In every row are 6 values ax, bx, ay, by, az, bz. Nodes with coordinate (kx,ky,kz) and (ax<=kx<=bx and ay<=ky<=by and az<=kz<=bz) will be ignored for the bars. Theses nodes are in nodes. But no bar will be connect to these nodes.

mountingarea [numpy.array] is a matrix describing areas where the nodes are fixed. (optional) In every row are 6 values ax, bx, ay, by, az, bz. Nodes with coordinate (kx,ky,kz) and (ax<=kx<=bx and ay<=ky<=by and az<=kz<=bz) are considered as fixed nodes. If mountingarea is denoted also mounting will be returned. The paarameter mounting will be ignored. But the affected bars will be ignored.

maxstablaenge [float] Bars which are longer than maxstablaenge will be ignored.

Returns:

nodes [numpy.array] is a matrix to represent all nodal points with:

- nodes(k,:) are the nodal points.
- nodes(:,1) are the x-coordinates of the nodal points.
- nodes(:,2) are the y-coordinates of the nodal points.
- nodes(:,3) are the z-coordinates of the nodal points.

bars [numpy.array] is a vector to represent all trusses:

- bars(s,:) are numbers of the trusses.
- bars(:,1) is the number of the first nodal point.
- bars(:,2) is the number of the second nodal point.

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed. (optional) It will will be returned only if mountingarea is denoted.

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`ttrr.`**`ttrr`**(*dim*, *nodes*, *bars*, *mounting*, *loads*, *smax*, *name='test'*, *sigmamin=-100000000.0*, *sigmamax=100000000.0*, *density=2700.0*, *redundancy=0*, *opt=0*, *undisturbed_forces=0*, *sort=0*, *zlb=0*, *zub=0*, *scale_sigma=-1.0*, *scale_forces=-1.0*)

[s,w,lengths] = ttrr(dim,nodes,bars,mounting,loads,smax,name='test', sigmamin=-1e8,sigmamax=1e8,density=2.7e3,redundancy=0,

opt=0,undisturbed_forces=0,sort=0,zlb=0,zub=0, scale_sigma=-1,scale_forces=-1)

Parameters:

dim [integer] dimension 2 or 3 of the design space

nodes [numpy.array] is a matrix to represent all nodal points with:

- nodes(k,:) are the nodal points.
- nodes(:,1) are the x-coordinates of the nodal points.
- nodes(:,2) are the y-coordinates of the nodal points.
- **nodes(:,3) are the z-coordinates of the nodal points.** (optional for dim = 2)

bars [numpy.array] is a vector to represent all trusses:

- bars(s,:) are numbers of the trusses.
- bars(:,1) is the number of the first nodal point.
- bars(:,2) is the number of the second nodal point.

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed.

loads [scipy.sparse.lil_matrix] are the load forces.

loads(:,k) are the forces for the k-th load case

smax [float] is the maximal allowed cross section of a bar. If you do not need it, set it to a huge value — e. g. 1.

name [string] is a name for the problem. The result is saved in name.tar.bz2 in the current working directory.

sigmamin [float] lower limit for the allowed stress, e.g. compressive strength (e.g. -1e8 Pa for aluminium or -4e8 Pa for some steel)

sigmamax [float] upper limit for the allowed stress, e.g. tensile strength (e.g. 1e8 Pa for aluminium or 4e8 Pa for some steel)

density [float] density (e.g. 2.7e3 kg/m^3 for aluminium or 7e3 kg/m^3 for some steel)

redundancy [integer] describes the redundancy. If redundancy != 0 this function is not the right one. `ttrr_redundant()` will be called.

opt [integer] choose the optimization routine: (optional)

- 0 glp_simplex (default, iff redundancy == 0)
- 1 glp_interior (only for redundancy == 0)
- **2-6 glp_mip (only for redundancy != 0, therefore not in this** function) see `ttrr_redundant()`

undisturbed_forces [numpy.array] is not used. It will be saved to the problem data.

sort [integer] describes a feature: (optional)

- 0 not used (default)
- **1 additional constraints will be added to the optimization** problem. This is only useful for redundancy != 0. see `ttrr_redundant()`

zlb [float] is a lower bound for the objective function. As we are minimizing the result must not the best solution. Only values greater zero are considered. (optional)

zub [float] is a upper bound for the objective function. As we are minimizing the optimization problem may be not solvable. Only values greater zero are considered. (optional)

scale_sigma [float]

scales sigmamin and sigmamax for the calculation. (optional)

- < 0.0 no scaling will be done (default)
- = 0.0 autoscaling will be done
- > 0.0 scaling will be done with this value

scale_forces [float]

scales loads for the calculation. (optional)

- < 0.0 no scaling will be done (default)
- = 0.0 autoscaling will be done
- > 0.0 scaling will be done with this value

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ttrr.**ttrr_redundant**(*dim*, *nodes*, *bars*, *mounting*, *loads*, *smax*, *name='test'*, *sigmamin=-100000000.0*, *sigmamax=100000000.0*, *density=2700.0*, *redundancy=0*, *opt=2*, *undisturbed_forces=0*, *sort=0*, *zlb=0*, *zub=0*, *scale_sigma=-1.0*, *scale_forces=-1.0*)

[s,w,exitflag,zeit,lengths] = ttrr_redundant(dim,nodes,bars,mounting, loads,smax,name='test',sigmamin=-1e8,

sigmamax=1e8,density=2.7e3,redundancy=0, opt=2,undisturbed_forces=0,sort=0, zlb=0,zub=0, scale_sigma=-1,scale_forces=-1)

Parameters:

dim [integer] dimension 2 or 3 of the design space

nodes [numpy.array] is a matrix to represent all nodal points with:

- nodes(k,:) are the nodal points.
- nodes(:,1) are the x-coordinates of the nodal points.
- nodes(:,2) are the y-coordinates of the nodal points.
- **nodes(:,3) are the z-coordinates of the nodal points.** (optional for dim = 2)

bars [numpy.array] is a vector to represent all trusses:

- bars(s,:) are numbers of the trusses.
- bars(:,1) is the number of the first nodal point.
- bars(:,2) is the number of the second nodal point.

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed.

loads [scipy.sparse.lil_matrix] are the load forces.

loads(:,k) are the forces for the k-th load case

smax [float] is the maximal allowed cross section of a bar. If you do not need it, set it to a huge value — e. g. 1. It is nessecary for the interaction of the integer values and the cross section values (for redundancy != 0).

name [string] is a name for the problem. The result is saved in name.tar.bz2 in the current working directory.

sigmamin [float] lower limit for the allowed stress, e.g. compressive strength (e.g. -1e8 Pa for aluminium or -4e8 Pa for some steel)

sigmamax [float] upper limit for the allowed stress, e.g. tensile strength (e.g. 1e8 Pa for aluminium or 4e8 Pa for some steel)

density [float] density (e.g. 2.7e3 kg/m^3 for aluminium or 7e3 kg/m^3 for some steel)

redundancy [integer] describes the redundancy. If redundancy == 0 this function is not the right one. `ttrr()` will be called. A value greater 0 describes a 1/redundancy redundancy — therefore redundancy >= 2 makes sense. A value lower 0 descirbes a 1-1/redundancy redundancy — therefore redundancy <= 2 makes sense. Obviosly 1/2 = 1-1/2 are the same redundancy. But the problem and the calculation are different. If there is no error the results are equal.

opt [integer] choose the optimization routine: (optional)

- 0 glp_simplex (only for redundancy == 0)
- 1 glp_interior (only for redundancy == 0)
- 2-6 glp_mip (for redundancy != 0)
  - 2 do not search greedy-solution (default)
  - 3 search greedy-solution and go on
  - 4 search greedy-solution, write greedy solution and go on
  - 5 search greedy-solution and stop
  - **6 search in parallel with 2 and 3 and stop** after solution is found with one method

undisturbed_forces [numpy.array] is not used. It will be saved to the problem data.

sort [integer] describes a feature: (optional)

- 0 not used (default)
- **1 additional constraints will be added to the optimization** problem. This is only useful for redundancy != 0. If s_i describes a vector of the cross section of the i-th part structure the constraints are l^T s_i <= l^T s_{i+1} for adequate i with l the vector of the lengths of all bars.

zlb [float] is a lower bound for the objective function. As we are minimizing the result must not the best solution. Only values greater zero are considered.

zub [float] is a upper bound for the objective function. As we are minimizing the optimization problem may be not solvable. Only values greater zero are considered.

scale_sigma [float]

scales sigmamin and sigmamax for the calculation. (optional)

- < 0.0 no scaling will be done (default)
- = 0.0 autoscaling will be done
- > 0.0 scaling will be done with this value

scale_forces [float]

scales loads for the calculation. (optional)

- < 0.0 no scaling will be done (default)
- = 0.0 autoscaling will be done
- > 0.0 scaling will be done with this value

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`ttrr.`**`ttrr_robust`**(*dim*, *nodes*, *bars*, *mounting*, *loads*, *smax*, *name='test'*, *sigmamin=-100000000.0*, *sigmamax=100000000.0*, *density=2700.0*, *redundancy=0*, *opt=0*, *undisturbed_forces=0*, *p=0.1*, *art=0*, *sort=0*, *zlb=0*, *zub=0*, *scale_sigma=-1.0*, *scale_forces=-1.0*)

[s,w,exitflag,zeit,lengths] = ttrr_robust(dim,nodes,bars,mounting, loads,smax,name='test',sigmamin=-1e8, sigmamax=1e8,density=2.7e3,redundancy=0, opt=0,undisturbed_forces=0,p=0.1,art=0, sort=0,zlb=0,zub=0, scale_sigma=-1,scale_forces=-1)

Parameters:

dim [integer] dimension 2 or 3 of the design space

nodes [numpy.array] is a matrix to represent all nodal points with:

- nodes(k,:) are the nodal points.
- nodes(:,1) are the x-coordinates of the nodal points.
- nodes(:,2) are the y-coordinates of the nodal points.
- **nodes(:,3) are the z-coordinates of the nodal points.** (optional for dim = 2)

bars [numpy.array] is a vector to represent all trusses:

- bars(s,:) are numbers of the trusses.
- bars(:,1) is the number of the first nodal point.
- bars(:,2) is the number of the second nodal point.

mounting [numpy.array] are the numbers of the nodal points as for the coordinate direction which are fixed.

loads [scipy.sparse.lil_matrix] are the load forces.

loads(:,k) are the forces for the k-th load case

smax [float] is the maximal allowed cross section of a bar. If you do not need it, set it to a huge value — e. g. 1. It is nessecary for the interaction of the integer values and the cross section values (for redundancy != 0).

name [string] is a name for the problem. The result is saved in name.tar.bz2 in the current working directory.

sigmamin [float] lower limit for the allowed stress, e.g. compressive strength (e.g. -1e8 Pa for aluminium or -4e8 Pa for some steel)

sigmamax [float] upper limit for the allowed stress, e.g. tensile strength (e.g. 1e8 Pa for aluminium or 4e8 Pa for some steel)

density [float] density (e.g. 2.7e3 kg/m^3 for aluminium or 7e3 kg/m^3 for some steel)

redundancy [integer] describes the redundancy. After evaluating the special parameter the other functions are called. If redundancy == 0 the function `ttrr()` will be called and if redundancy != 0 the function `ttrr_redundant()` will be called.

opt [integer] choose the optimization routine: (optional)

- 0 glp_simplex (for redundancy == 0) (default)
- 1 glp_interior (for redundancy == 0)
- 2-6 glp_mip (for redundancy != 0)
  - 2 do not search greedy-solution
  - 3 search greedy-solution and go on
  - 4 search greedy-solution, write greedy solution and go on
  - 5 search greedy-solution and stop
  - **6 search in parallel with 2 and 3 and stop** after solution is found with one method

undisturbed_forces [numpy.array] If it is a scalar the perturbed forces are calculated with `compute_disturbed_forces()`. Otherwise loads are assumed to be the perturbed forces. The content of undisturbed_forces is not used. It will be saved to the problem data.

p [float] is a parameter for the calculation of perturbed forces. (optional) see `compute_disturbed_forces()`

art [integer] chooses the kind of the perturbation: (optional) see `compute_disturbed_forces()`

sort [integer] describes a feature: (optional)

- 0 not used (default)
- **1 additional constraints will be added to the optimization** problem. see `ttrr_redundant()`

zlb [float] is a lower bound for the objective function. As we are minimizing the result must not the best solution. Only values greater zero are considered.

zub [float] is a upper bound for the objective function. As we are minimizing the optimization problem may be not solvable. Only values greater zero are considered.

scale_sigma [float]

scales sigmamin and sigmamax for the calculation. (optional)

- < 0.0 no scaling will be done (default)
- = 0.0 autoscaling will be done
- > 0.0 scaling will be done with this value

scale_forces [float]

scales loads for the calculation. (optional)

- < 0.0 no scaling will be done (default)
- = 0.0 autoscaling will be done
- > 0.0 scaling will be done with this value

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Date: 2012-02-19 (last change)

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2.4 dependencies

Dependencies for ttrr.py and ttrr_tools.py:

- Python http://www.python.org/
- NumPy http://numpy.scipy.org/ http://pypi.python.org/pypi/numpy
- SciPy http://www.scipy.org/ http://pypi.python.org/pypi/scipy
  - and also for NumPy/SciPy: BLAS and LAPACK libraries

Dependencies for ttrr and ttrr.cc, respectively:

- GLPK http://www.gnu.org/software/glpk/

Runtime dependencies for ttrr_tools.py to get graphical output (picture, movie, animation) you need some of the following programs:

- gnuplot http://www.gnuplot.info/
- POV-Ray http://www.povray.org/
- png2yuv from MJPEG http://mjpeg.sourceforge.net/
- ffmpeg2theora http://www.v2v.cc/~j/ffmpeg2theora/
- png2theora from libtheora http://www.theora.org
- FFmpeg http://ffmpeg.org/
- APNG Assembler http://sourceforge.net/projects/apngasm/
- convert from ImageMagick http://www.imagemagick.org/

At least for pictures you need gnuplot and/or POV-Ray. The frames for all movie and animation output are rendered with POV-Ray.

2.5 References:

- Mohr, Daniel P. ; Stein, Ina ; Matzies, Thomas ; Knapek, Christina A.: Redundant robust topology optimization of truss. In: Optimization and Engineering (2013), 1-28. http://dx.doi.org/10.1007/s11081-013-9241-7
- Mohr, Daniel P. ; Stein, Ina ; Matzies, Thomas ; Knapek, Christina A.: Robust Topology Optimization of Truss with regard to Volume. In: arXiv - Mathematics, Optimization and Control (2012). http://arxiv.org/abs/1109.3782v2
- Mohr, Daniel P.: Redundante Topologieoptimierung. Neubiberg, Universitaet der Bundeswehr Muenchen, Fakultaet fuer Luft- und Raumfahrttechnik, Diss., Dezember 2011. http://nbn-resolving.de/urn/resolver.pl?urn=urn:nbn:de:bvb:706-2664

2.6 copyright + license

Author: Daniel Mohr.

Date: 2013-11-09 (last change).

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CHAPTER

THREE

TTRR_TOOLS.PY

ttrr_tools.py is a command line tool. You can get the help as usual:

```
$ ttrr_tools.py -h

usage: ttrr_tools.py [-h] -f file [file ...] [--new] [--version]
                     [--png_gnuplot] [--pdf_gnuplot] [--gnuplot_cylinder]
                     [--png_pov] [--ogv]
                     [-ogv_convert {ffmpeg2theora,png2theora,ffmpeg}] [--apng]
                     [--mng] [--gif] [--yuv] [--all_bars]
                     [--coordinate_system] [--small_coordinate_system]
                     [--show_result_table] [--show_results]
                     [-hist {log_asciiart,numbers,asciiart,all,none}]
                     [-ascii_hist_l lines] [-ascii_hist_c cols]
                     [--save_module] [--save_mat] [-x pixels] [-y pixels]
                     [-xm pixels] [-ym pixels] [-frames n] [-framerate n]
                     [-pov_angle a] [-pov_angle_m a] [-minfact mf]

ttrr - TrussTopologyRobustRedundant: ttrr is a software package to do topology
optimization of a truss with respect to robustness and redundancy. For more help
type "pydoc ttrr"

optional arguments:
  -h, --help            show this help message and exit
  -f file [file ...]    file(s) to process
  --new                 create only new pictures/movies. do not overwrite
                        existing ones.
  --version             show program's version number and exit
  --png_gnuplot         create png-pictures with gnuplot
  --pdf_gnuplot         create pdf-pictures with gnuplot
  --gnuplot_cylinder    Use gnuplot with creating cylinders as bars. This is
                        slow.
  --png_pov             create png-pictures with povray
  --ogv                 create ogv-movies with povray and a converter
  -ogv_convert {ffmpeg2theora,png2theora,ffmpeg}
                        Set the converter to generate the movie from the png-
                        pictures, which povray creates. To create a movie with
                        ffmpeg2theora it needs png2yuv. default: ffmpeg2theora
  --apng                create apng-animation with apngasm
  --mng                 create mng-animation with convert
  --gif                 create gif-animation with convert
  --yuv                 create YUV4MPEG stream format with png2yuv. This is an
                        uncompressed movie-stream. It is only useful for
                        further conversions.
  --all_bars            create pictures with all bars (structural universe)
  --coordinate_system   create pictures with a coordinate system
  --small_coordinate_system
                        create pictures with a small coordinate system
  --show_result_table   show results in a table
```

```
  --show_results        show results
  -hist {log_asciiart,numbers,asciiart,all,none}
                        Show in results histogram(s) of bar cross sections of
                        this type. default: log_asciiart
  -ascii_hist_l lines   Set in asciiart histogram the numbers of lines.
                        (default: 3)
  -ascii_hist_c cols    Set in asciiart histogram the numbers of cols.
                        (default: 100)
  --save_module         save with scipy.io.save_as_module data from file(s) to
                        "{.dat,.dir,.py}"-files
  --save_mat            save with scipy.io.savemat data from file(s) to
                        ".mat"-file(s)
  -x pixels             x resolution of the pictures (default: 1000)
  -y pixels             y resolution of the pictures (default: 1000)
  -xm pixels            x resolution of the movies (default: 800)
  -ym pixels            y resolution of the movies (default: 600)
  -frames n             the movie will be created from n frames (default: 250)
  -framerate n          set the framerate. the movie will be created with n
                        frames per second (default: 25)
  -pov_angle a          For rendering pictures by povray the view angle is set
                        to a (default: 33)
  -pov_angle_m a        For rendering animations by povray the view angle is
                        set to a (default: 40)
  -minfact mf           a truss with a cross section less than mf*max(s) is
                        ignored. max(s) is the maximum over all truss cross
                        sections. if mf=0 no truss is ignored. (default: 1e-6)

Author: Daniel Mohr
Date: 2013-11-09
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```

CHAPTER

FOUR

EXAMPLE: BEN-TAL_NEMIROVSKI_PRIME_EXAMPLE_1997

Contents

This example shows you how `ttrr` is used. Some parts are unnecessary, but show the content/meaning of the concerning values or show same features/tools of `ttrr`. It's an example file!

This example is also available as `batch_ben-tal_nemirovski_prime_example_1997.py`.

This example describes the prime example of a truss from: Aharon Ben-Tal and Arkadi Nemirovski. Robust truss topology design via semidefinite programming. SIAM J. Optim., 7(4):991-1016, November 1997. ISSN 1052-6234 (print), 1095-7189 (electronic). http://dx.doi.org/10.1137/S1052623495291951 .

First start python and import necessary modules:

```
>>> import ttrr,numpy,scipy # we need ttrr and other great packages
```

4.1 design space

To define the design space:

```
>>> dim = 2 # dimension of the design space
>>> nelx = 3 # number of nodes in x direction
>>> nely = 2 # number of nodes in y direction
>>> kn = nelx*nely # number or total nodes
```

The definition of the grid:

```
>>> gx0 = 0.0 # meter
>>> gx1 = 2.0 # meter
>>> gy0 = 0.0 # meter
>>> gy1 = 1.0 # meter
```

Now we have a design space of nelx*nely=3*2=6 nodes. The design space is a rectangle of the points (x,y) with gx0 <= x <= gx1 and gy0 <= y <= gy1. We need the coordinates of all nodes:

```
>>> nodes = numpy.array([[ 0.0 , 0.0 ], # node 0 with the coordinates (0.0,0.0) = nodes[0,:]
                         [ 0.0 , 1.0 ], # node 1 with the coordinates (0.0,1.0) = nodes[1,:]
                         [ 1.0 , 0.0 ], # ...
                         [ 1.0 , 1.0 ],
                         [ 2.0 , 0.0 ],
                         [ 2.0 , 1.0 ]],dtype=numpy.float64) # the coordinates are float64 (doubl
```

We need a description of all trusses (from every node to every other one):

```
>>> bars = numpy.array([[0,2], # truss from node 0 with nodes[0,:] to node 2 with nodes[2,:]
                        [0,1], # truss from node 0 with nodes[0,:] to node 1 with nodes[1,:]
                        [0,3], # ...
                        [0,5],
                        [1,3],
                        [2,4],
                        [1,2],
                        [2,3],
                        [2,5],
                        [3,5],
                        [1,4],
                        [3,4],
                        [4,5]],dtype=numpy.uint16) # the number of nodes are uint16 (16-bit int
```

We do not need a truss from 0 to 4 or from 1 to 5, because it is the same as from 0 to 2 and from 2 to 4 or from 1 to 3 and from 3 to 5, respectively.

We can get this from, too (This is handy if you have more than 6 nodes!):

```
>>> [nodes,bars] = ttrr.barsnodesongrid2d(gx0,gx1,gy0,gy1,nelx,nely)
```

4.2 bearings

We want to fix the node 0 and 1 in every direction:

- 0 for node 0 in x direction
- 1 for node 0 in y direction
- 2 for node 1 in x direction
- 3 for node 1 in x direction

```
>>> mounting = numpy.array([0,1,2,3],dtype=numpy.uint16)
```

The truss from node 0 to node 1 is on both ends fixed. So this truss do not make sense. To reduce the number of design variables we can do it without this truss:

```
>>> [nodes,bars] = ttrr.barsnodesongrid2d(gx0,gx1,gy0,gy1,nelx,nely,mounting=mounting)
13  potenzielle Staebe gefunden; beruecksichtige 4 Lager
es bleiben 12 Staebe, die nicht nur gelagert sind
```

4.3 load cases

We have only 1 load case, but with “some” forces on different nodes:

```
>>> loads = scipy.sparse.lil_matrix((dim*kn,1),dtype=numpy.float64)
>>> loads[4,0] = -10000 # force in node 2 in x direction means 4 in the load case 0 with a value
>>> loads[7,0] = 10000 # force in node 3 in y direction means 7 in the load case 0 with a value o
>>> loads[9,0] = -10000 # force in -y direction with the magnitude 10000 N in node 4
>>> loads[10,0] = 10000 # force in x direction with the magnitude 10000 N in node 5
```

The magnitude of each force is only a scalar factor in the optimization problem. The commanding spot is the proportion of the forces among each other. The same holds for the material parameters.

4.4 calculation

We need a few parameters for the calculation:

```
>>> smax = 1 # the maximum allowed cross section in meter (here: not really a bound)
>>> name = "ben-tal_nemirovski_prime_example" # a name for the output
```

Some material parameter (The compressive strength sigmamin and the tensile strength sigmamax are only scalar factors in the calculation. The density density is only used to show the mass of the result.):

```
>>> sigma = 1e8 # Pa = 100 MPa = 100 N / mm^2 (yield strength for aluminium)
>>> sigmamin = - sigma # compressive strength
>>> sigmamax = sigma # tensile strength
>>> density = 2.7e3 # kg / m^3 (density for aluminium)
```

Now we can calculate the result:

```
>>> [s,w,lengths] = ttrr.ttrr(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sigmamin,sigma
###### start ttrr ######
tmpdir: /tmp/ttrr_tmp__kA2eC
speichere nach ben-tal_nemirovski_prime_example.tar.bz2
### --- ### --- ### --- ### --- ### --- ### --- ###
starte: ttrr_calculate /tmp/ttrr_tmp__kA2eC
### ttrr ### gestartet ###
lese /tmp/ttrr_tmp__kA2eC/dim.bin
lese /tmp/ttrr_tmp__kA2eC/nodes.bin
lese /tmp/ttrr_tmp__kA2eC/bars.bin
lese /tmp/ttrr_tmp__kA2eC/mounting.bin
lese /tmp/ttrr_tmp__kA2eC/loads.bin
lese /tmp/ttrr_tmp__kA2eC/smax.bin
lese /tmp/ttrr_tmp__kA2eC/info.txt
lese /tmp/ttrr_tmp__kA2eC/redundancy.bin
lese /tmp/ttrr_tmp__kA2eC/opt.bin
lese /tmp/ttrr_tmp__kA2eC/sort.bin
lese /tmp/ttrr_tmp__kA2eC/sigmamin.bin
lese /tmp/ttrr_tmp__kA2eC/sigmamax.bin
lese /tmp/ttrr_tmp__kA2eC/density.bin
lese /tmp/ttrr_tmp__kA2eC/zlb.bin
lese /tmp/ttrr_tmp__kA2eC/zub.bin
ben-tal_nemirovski_prime_example: dim = 2 ; smax = 1.000000e+00 ; redundancy = 0 ; opt = 0 ; sort
sigmamin = -1.000000e+08 ; sigmamax = 1.000000e+08 ; density = 2.700000e+03
6 Knoten ; 12 Staebe ; 4 Lager ; 1 Lastfaelle
calculate geometry matrix and length of bars...geometry matrix (36 non-zero elements) and length
schreibe /tmp/ttrr_tmp__kA2eC/lengths.bin
schreibe /tmp/ttrr_tmp__kA2eC/lengths.txt
schreibe /tmp/ttrr_tmp__kA2eC/C.bin
schreibe /tmp/ttrr_tmp__kA2eC/C.txt
berechne freedofs...freedofs berechnet
schreibe /tmp/ttrr_tmp__kA2eC/freedofs.bin
```

```
schreibe /tmp/ttrr_tmp__kA2eC/freedofs.txt
beschraenke Geometriematrix (12x12) auf freie Knoten...Geometriematrix (8x12) auf freie Knoten bes
schreibe /tmp/ttrr_tmp__kA2eC/C_freedofs.bin
schreibe /tmp/ttrr_tmp__kA2eC/C_freedofs.txt
beschraenke Kraefte (12x1) auf freie Knoten...Kraefte (8x1) auf freie Knoten beschraenkt
schreibe /tmp/ttrr_tmp__kA2eC/loads_freedofs.bin
schreibe /tmp/ttrr_tmp__kA2eC/loads_freedofs.txt
erstelle Optimierungsproblem
berechne Zielfunktion...Zielfunktion berechnet
berechne Gleichungen...8 Gleichungen fuer 24 Unbekannte berechnet (26 Nicht-Null-Elemente)
schreibe /tmp/ttrr_tmp__kA2eC/Aeq.bin
schreibe /tmp/ttrr_tmp__kA2eC/Aeq.txt
schreibe /tmp/ttrr_tmp__kA2eC/beq.bin
schreibe /tmp/ttrr_tmp__kA2eC/beq.txt
berechne Unleichungen...24 Unleichungen fuer 24 Unbekannte berechnet (48 Nicht-Null-Elemente)
schreibe /tmp/ttrr_tmp__kA2eC/A.bin
schreibe /tmp/ttrr_tmp__kA2eC/A.txt
schreibe /tmp/ttrr_tmp__kA2eC/b.bin
schreibe /tmp/ttrr_tmp__kA2eC/b.txt
berechne lb und ub...lb und ub berechnet
schreibe /tmp/ttrr_tmp__kA2eC/lb.bin
schreibe /tmp/ttrr_tmp__kA2eC/lb.txt
schreibe /tmp/ttrr_tmp__kA2eC/ub.bin
schreibe /tmp/ttrr_tmp__kA2eC/ub.txt
A[23,23] = -1.000000e+08
erstelle Gesamtmatrix (32x24) aus 74 Nicht-Null-Elementen...erstellt; lade Gesamtmatrix...Gesamtma
Writing problem data to '/tmp/ttrr_tmp__kA2eC/ben-tal_nemirovski_prime_example.glpk.gz'...
69 lines were written
Writing problem data to '/tmp/ttrr_tmp__kA2eC/ben-tal_nemirovski_prime_example.prob.gz'...
141 lines were written
berechne...
opt == 0: glp_simplex
GLPK Simplex Optimizer, v4.45
32 rows, 24 columns, 74 non-zeros
Preprocessing...
32 rows, 24 columns, 74 non-zeros
Scaling...
 A: min|aij| =  4.472e-01  max|aij| =  1.000e+08  ratio =  2.236e+08
GM: min|aij| =  8.286e-01  max|aij| =  1.207e+00  ratio =  1.456e+00
EQ: min|aij| =  7.071e-01  max|aij| =  1.000e+00  ratio =  1.414e+00
Constructing initial basis...
Size of triangular part = 32
      0: obj =   0.000000000e+00  infeas =  1.740e+09 (0)
*    26: obj =   1.300000000e-03  infeas =  3.150e-12 (0)
*    29: obj =   8.000000000e-04  infeas =  8.078e-28 (0)
OPTIMAL SOLUTION FOUND
Writing basic solution to '/tmp/ttrr_tmp__kA2eC/ben-tal_nemirovski_prime_example.out.gz'...
Writing basic solution to '/tmp/ttrr_tmp__kA2eC/ben-tal_nemirovski_prime_example.sol.gz'...
58 lines were written
Zielfunktionswert: 0.000800
Gesamtvolumen: 0.000800
Gesamtgewicht: 2.160000
schreibe /tmp/ttrr_tmp__kA2eC/w.bin
schreibe /tmp/ttrr_tmp__kA2eC/w.txt
schreibe /tmp/ttrr_tmp__kA2eC/s.bin
schreibe /tmp/ttrr_tmp__kA2eC/s.txt
schreibe /tmp/ttrr_tmp__kA2eC/objectivefunctionvalue.bin
schreibe /tmp/ttrr_tmp__kA2eC/objectivefunctionvalue.txt
gebe Speicher wieder frei
### ttrr ### beendet ###
### --- ### --- ### --- ### --- ### --- ### --- ###
speichere nach ben-tal_nemirovski_prime_example.tar.bz2
###### end ttrr ######
```

As in "Aharon Ben-Tal and Arkadi Nemirovski. Robust truss topology design via semidefinite programming. SIAM J. Optim., 7(4):991-1016, November 1997. ISSN 1052-6234 (print), 1095-7189 (electronic). http://dx.doi.org/10.1137/S1052623495291951 ." we can get a robust solution; but here with a linear program as described in "Mohr, Daniel P. ; Stein, Ina ; Matzies, Thomas ; Knapek, Christina A.: Robust Topology Optimization of Truss with regard to Volume. In: arXiv - Mathematics, Optimization and Control (2012). http://arxiv.org/abs/1109.3782" (This time the output is omitted.):

```
>>> name = "ben-tal_nemirovski_prime_example_robust" # a name for the output
>>> [s,w,lengths] = ttrr.ttrr_robust(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sigmam
```

More explicitly we can calculate (or even define) the perturbed forces:

```
>>> forces_disturbed = ttrr.compute_disturbed_forces(loads,dim,0.1,art=0)
###### compute_disturbed_forces
(kkn,nl) = (12,1); Knotenzahl: 6; Dimension: 2
###### compute_disturbed_forces
(kkn,nl) = (12,1); Knotenzahl: 6; Dimension: 2
######
max_force_undisturbed = 20000.000000
max_force_disturbed = 20784.609691
skaliere Kraefte
max_force_disturbed = 20000.000000
######
```

And do the computation with these (The output is omitted.):

```
>>> [s,w,lengths] = ttrr.ttrr(dim,nodes,bars,mounting,forces_disturbed,smax,name=name,sigmamin=si
```

For the same problem we can calculate a (1/2)-redundant solution as described in "Mohr, Daniel P. ; Stein, Ina ; Matzies, Thomas ; Knapek, Christina A.: Redundant robust topology optimization of truss. In: Optimization and Engineering (2013), 1-28. http://dx.doi.org/10.1007/s11081-013-9241-7" and in "Mohr, Daniel P.: Redundante Topologieoptimierung. Neubiberg, Universitaet der Bundeswehr Muenchen, Fakultaet fuer Luft- und Raumfahrttechnik, Diss., Dezember 2011. http://nbn-resolving.de/urn/resolver.pl?urn=urn:nbn:de:bvb:706-2664":

```
>>> name = "ben-tal_nemirovski_prime_example_redundant" # a name for the output
>>> [s,w,lengths] = ttrr.ttrr_redundant(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sig
```

4.5 visualization

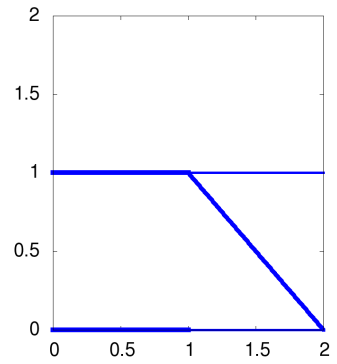
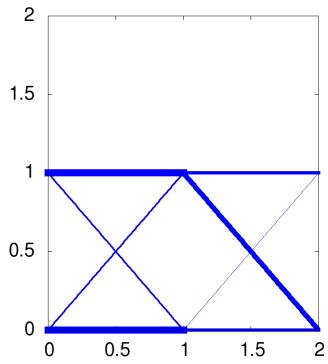
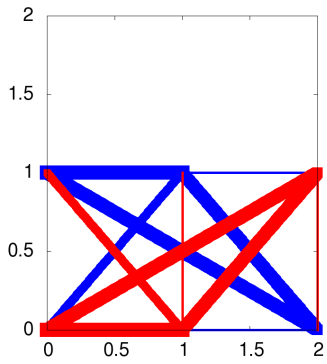
To get pictures, we can use *ttrr_tools.py* on the command line:

```
$ ttrr_tools.py --png_gnuplot -f ben-tal_nemirovski_prime_example.tar.bz2 ben-tal_nemirovski_prime
```

Or for short:

```
$ ttrr_tools.py --png_gnuplot -f ben-tal_nemirovski_prime_example*.tar.bz2
```

Now we get the example pictures:

optimal solution	robust solution	redundant solution
		

I think these pictures are the best way for our human being to understand the topologies. Keep in mind that the topology is not the sizing of parts of the structure. But the calculation needs the sizing values to benchmark the topology. So it is impossible to get a topology of a structure without the sizing values.

4.6 conclusion

The degree of freedom for this problem is very small. This means: The set of admissible solutions is small. So the optimization routine has only a small choice. The consequence is that in particular the result of the redundant solution is poor. But this file shows the functionality of ttrr and also the basic concept of robustness and redundancy. I hope you get the point.

CHAPTER

FIVE

EXAMPLE: 2_LOAD_CASES_3_DIM

Contents

This is a 3 dimensional example to show `ttrr`.

This example is also available as `batch_2_load_cases_3_dim.py`.

First start python and import necessary modules:

```
>>> import ttrr,numpy,scipy
```

5.1 design space

To define the design space:

```
>>> dim = 3 # dimension of the design space
>>> nelx = 3 # number of nodal points in x-direction
>>> nely = nelx # number of nodal points in y-direction
>>> nelz = nelx # number of nodal points in z-direction
```

To let you easily change the design space, we use here a calculation to define the grid. The measurement unit is meter:

```
>>> n = max(nelx,nely,nelz)
>>> gx0 = 0.0
>>> gx1 = (nelx-1)*1.0/(n-1)
>>> gy0 = 0.0
>>> gy1 = (nely-1)*1.0/(n-1)
>>> gz0 = 0.0
>>> gz1 = (nelz-1)*1.0/(n-1)
```

5.2 bearings

We want to fix all nodes on the ground. We can describe these area (All nodes will become fixed bearings):

```
>>> mountingarea = numpy.array([
...     [ gx0 , gx1 , gy0 , gy1 , gz0 , gz0 ]
...     ])
>>> mountingarea[:,[0,2,4]] = mountingarea[:,[0,2,4]] - 1e-6
>>> mountingarea[:,[1,3,5]] = mountingarea[:,[1,3,5]] + 1e-6
```

5.3 get the parameter

Now we can get the parameter for the calculation from barsnodesongrid3d. Here we can also use the optional parameter maxstablaenge to decide that bars longer than maxstablaenge should be ignored:

```
>>> maxstablaenge = 3 # maximum allowed length of a bar
>>> [nodes,bars,mounting] = ttrr.barsnodesongrid3d(gx0,gx1,gy0,gy1,gz0,gz1,nelx,nely,nelz,mountinc
mounting wird aus 1 Zeilen von mountingarea erstellt
9 Knoten sind gelagert; 27 Lager
302 potenzielle Staebe gefunden; beruecksichtige 27 Lager
es bleiben 274 Staebe, die nicht nur gelagert sind
274 potenzielle Staebe; beruecksichtige maximale Stablaenge 3.000000
es bleiben 274 Staebe, die kuerzer sind als 3.000000
```

We can look at the Parameter:

```
>>> nodes
array([[ 0. ,  0. ,  0. ],
       [ 0. ,  0.5,  0. ],
       [ 0. ,  1. ,  0. ],
       [ 0.5,  0. ,  0. ],
       [ 0.5,  0.5,  0. ],
       [ 0.5,  1. ,  0. ],
       [ 1. ,  0. ,  0. ],
       [ 1. ,  0.5,  0. ],
       [ 1. ,  1. ,  0. ],
       [ 0. ,  0. ,  0.5],
       [ 0. ,  0.5,  0.5],
       [ 0. ,  1. ,  0.5],
       [ 0.5,  0. ,  0.5],
       [ 0.5,  0.5,  0.5],
       [ 0.5,  1. ,  0.5],
       [ 1. ,  0. ,  0.5],
       [ 1. ,  0.5,  0.5],
       [ 1. ,  1. ,  0.5],
       [ 0. ,  0. ,  1. ],
       [ 0. ,  0.5,  1. ],
       [ 0. ,  1. ,  1. ],
       [ 0.5,  0. ,  1. ],
       [ 0.5,  0.5,  1. ],
       [ 0.5,  1. ,  1. ],
       [ 1. ,  0. ,  1. ],
       [ 1. ,  0.5,  1. ],
       [ 1. ,  1. ,  1. ]])
>>> bars
array([[ 0,  9],
       [ 0, 12],
       [ 0, 21],
```

```
       [ 0, 15],
       [ 0, 10],
       [ 0, 19],
       [ 0, 11],
       [ 0, 13],
       [ 0, 22],
       [ 0, 14],
       [ 0, 23],
       [ 0, 16],
       [ 0, 25],
       [ 0, 17],
       [ 9, 18],
       [ 9, 10],
       ...
       [17, 22]], dtype=uint16)
>>> mounting
array([ 0,  1,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 14, 15, 16,
       17, 18, 19, 20, 21, 22, 23, 24, 25, 26])
>>> (kn,kr) = nodes.shape
>>> (sn,sr) = bars.shape
>>> (ln,) = mounting.shape
```

This means: We have kn=27 nodes and sn=274 bars. ln=27 node-directions are fixed. sr=2 is the number of nodes for a bar and kr=3 is the dimension of the design space.

5.4 load cases

The loads are stored in a sparse matrix. For every node we have dim directions. So we have dim*kn rows and 2 cols for the 2 load cases:

```
>>> loads = scipy.sparse.lil_matrix((dim*kn,2),dtype=numpy.float64) # 2 load cases
```

The number k1=19 of a node can be calculated from its coordinates. The direction r1=57 is in z-direction. The magnitude -100000.0 descibes that the direction is the negative z-direction:

```
>>> k1 = ttrr.nodenumberfromcoordinates(nodes,gx0,(gy0+gy1)/2,gz1) # position of the first force
>>> r1 = dim*k1 # direction of the first force
>>> loads[r1,0] = -100000.0 # Newton = ca. 1000 kg
>>> k1,r1
(19, 57)
```

The same for the other load case:

```
>>> k2 = ttrr.nodenumberfromcoordinates(nodes,gx1,(gy0+gy1)/2,gz1) # position of the second force
>>> r2 = dim*k2 # direction of the second force
>>> loads[r2,1] = 100000.0 # Newton = ca. 1000 kg
>>> k2,r2
(25, 75)
```

We can look at the result:

```
>>> print loads
  (57, 0)       -100000.0
  (75, 1)       100000.0
```

5.5 calculation

We need a few parameters for the calculation:

```
>>> smax = 1 # meter^2
>>> sigma = 1e8 # Pa = 100 MPa = 100 N / mm^2 (yield strength for aluminium)
>>> sigmamin = - sigma # compressive strength
>>> sigmamax = sigma # tensile strength
>>> density = 2.7e3 # kg / m^3 (density for aluminium)
>>> name="2_load_cases_3_dim"
```

Now we can do the classical topology optimization:

```
>>> [s,w,lengths] = ttrr.ttrr(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sigmamin,sigma
```

To calculate a redundant solution we need (This takes about 47 seconds on a computer with 3.6 GHz):

```
>>> redundancy = 3
>>> name="2_load_cases_3_dim_redundant"
>>> [s,w,lengths] = ttrr.ttrr_redundant(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sigm
```

5.6 visualization

To get pictures, we can use *ttrr_tools.py* on the command line. To get help:

```
$ ttrr_tools.py -h
```

We get quick and dirty pictures with gnuplot:

```
$ ttrr_tools.py --png_gnuplot -f 2_load_cases_3_dim*.tar.bz2
```

Nice pictures are rendered by povray:

```
$ ttrr_tools.py --png_pov -f 2_load_cases_3_dim*.tar.bz2
```

Because we are here viewing a 3 dimensional structure the 2 dimensional picture is insufficient. Therefore it is possible to get a movie animation:

```
$ ttrr_tools.py --ogv -f 2_load_cases_3_dim*.tar.bz2
```

If you only want a small animation you can do one of the following:

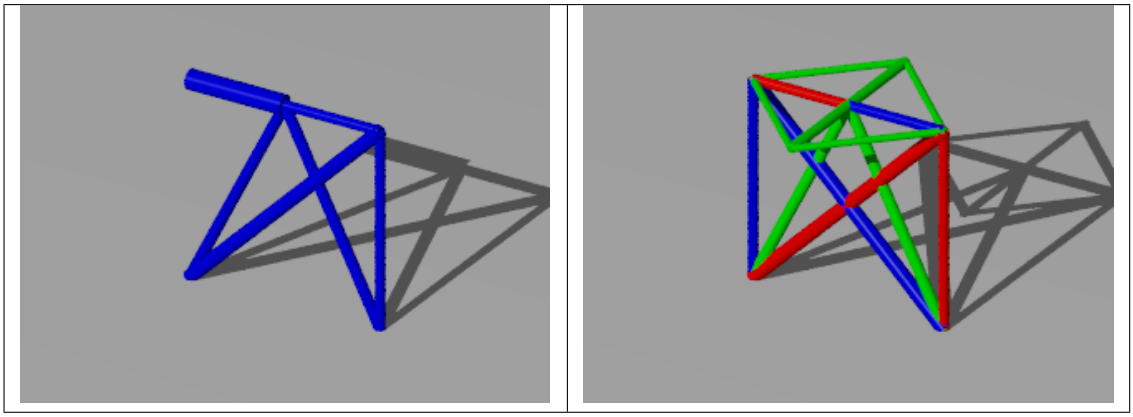
```
$ ttrr_tools.py --apng -xm 320 -ym 240 -f 2_load_cases_3_dim*.tar.bz2
$ ttrr_tools.py --mng -xm 320 -ym 240 -f 2_load_cases_3_dim*.tar.bz2
$ ttrr_tools.py --gif -xm 320 -ym 240 -f 2_load_cases_3_dim*.tar.bz2
```

You can also combine these options:

```
$ ttrr_tools.py -xm 320 -ym 240 -f 2_load_cases_3_dim*.tar.bz2 --png_pov --apng --mng --gif
```

The rendering by povray is time-consuming. Therefore it's done in parallel.

The above animation as apng:



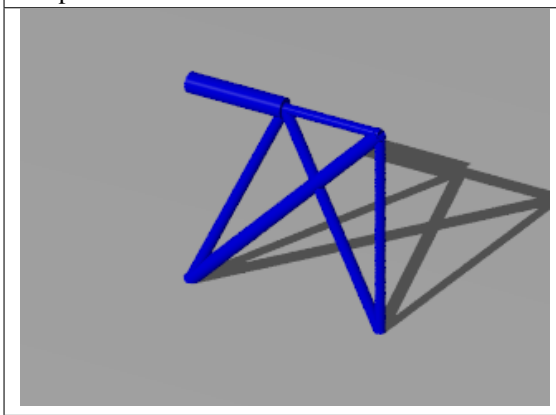
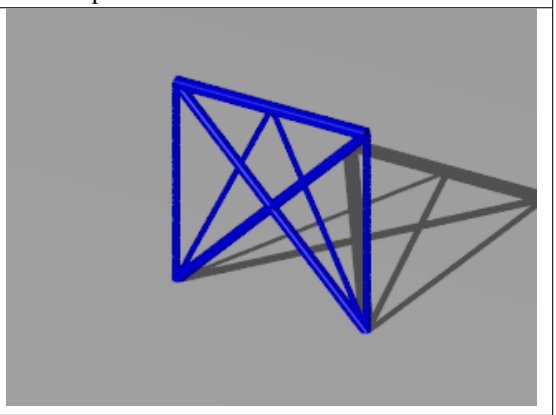
5.7 conclusion

Many people are wondering because the asymmetrical result. At last it's a symmetric problem. But we have a linear problem. Therefore there possibly exists more than one edge of the polytope as an optimal solution — the optimal value is the same. If we use an interior point algorithm we get a result between these edges.

The interior point algorithm of glpk has a big problem with scaling. Therefore we do a small scaling per hand (Note: This is only a scaling factor. The topology is the same.):

```
>>> name="2_load_cases_3_dim_interior"
>>> [s,w,lengths] = ttrr.ttrr(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sigmamin,sigma
```

Now we can look at the result:

simplex	interior point
	

I hope you get the picture of the 2 edges of the polytope in this problem.

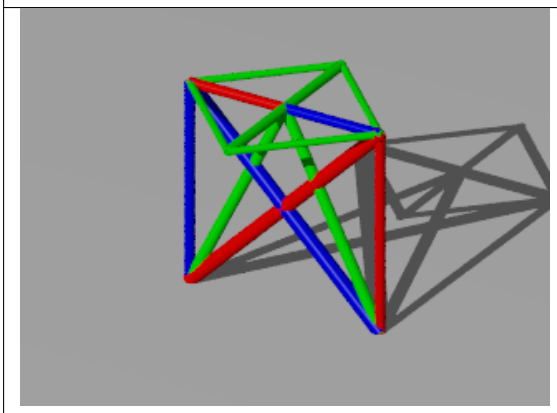
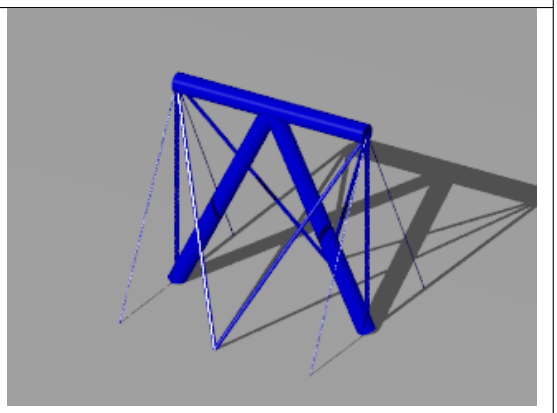
Another interesting point is the fact that we see a 2 dimensional structure in a 3 dimension space. Because the loads are only acting in one plane this is possible. Sure a small perturbation would destroy this behavior. Therefore it is not a practical solution. If you want a practical solution think of robust:

```
>>> name="2_load_cases_3_dim_robust"
>>> [s,w,lengths] = ttrr.ttrr_robust(dim,nodes,bars,mounting,loads,smax,name=name,sigmamin=sigmam
```

And an animation:

```
$ ttrr_tools.py -xm 320 -ym 240 -f 2_load_cases_3_dim_robust.tar.bz2 --png_pov --apng --mng --gif
```

The redundant solution is for a redundancy of 1/3. Every color represent a part of the structure. Every part is allow to drop out. The other parts are able to support the given loads. The interesting point is that the red or blue structure are not connected.

redundant	robust
	

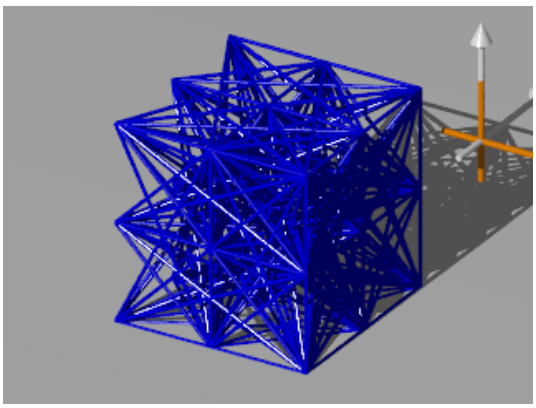
We can also compare the optimal values (This is the volume of the structures.):

optimal solution	redundant solution	optimal solution	robust solution
simplex	branch and bound	interior point	simplex
0.000875	0.001563	0.000875	0.001154

CHAPTER

SIX

EXAMPLE 3



This is a 3-dimensional example with the ground structure of 27 nodes $(a, b, c) \in \mathbb{R}^3$, $a, b, c \in \{1, 2, 3\}$ measured in meter and 274 potential bars between every two not fixed nodes – long bars which are located alongside several shorter bars are ignored. The supports are in the nodes $(1, a, b)$, $a, b \in \{1, 2, 3\}$ which are fixed in every direction. In the node $(3, 2, 1)$ acts a force of $4 \cdot 10^5$ N in the negative z-direction.

For a perturbation of $10\,\%$ in every direction of the force, and scaling to obtain the same maximal quantities as in the unperturbed case, our robust optimization method produces the robust optimal solutions. In contrast to the optimal solution the robust solution use the third dimension of the space.

6.1 Comparison of the volumes

	optimal	robust
volume:	6.0000e-03	6.5822e-03

Comparison of the volumes for different redundancies:

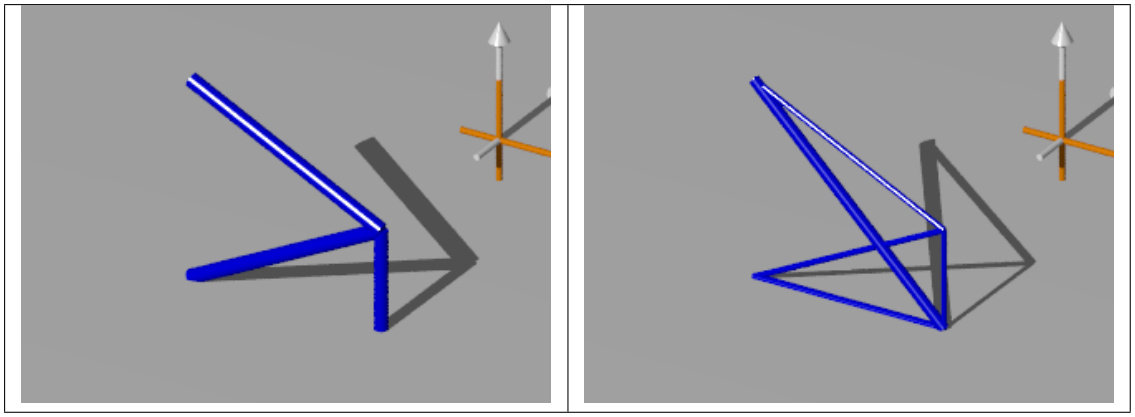
redundancy	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$1-\frac{1}{2}$	$1-\frac{1}{3}$	$1-\frac{1}{4}$
volume:	8.6667e-03	9.5000e-03	1.2000e-02	1.2000e-02	1.9000e-02	2.6000e-02

Comparison of the volumes for different redundancies and robustness:

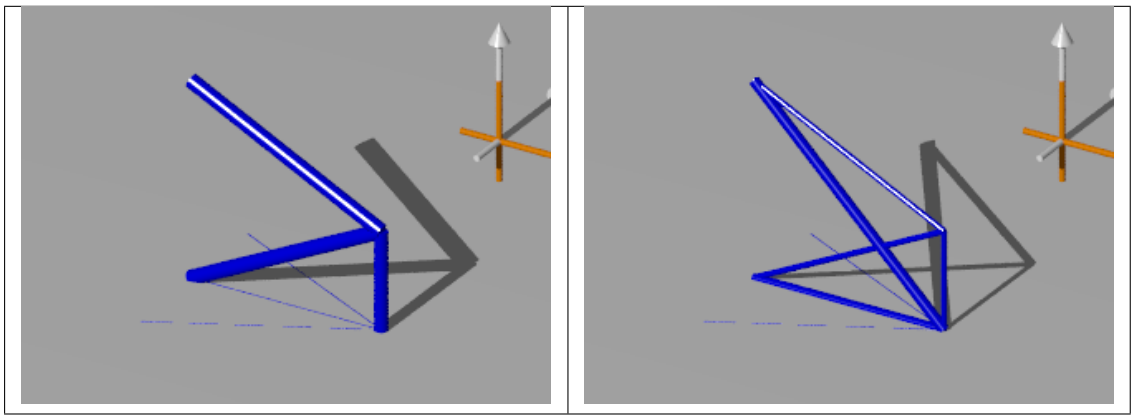
redundancy	$\frac{1}{3}$	$\frac{1}{2}$	$1-\frac{1}{2}$	$1-\frac{1}{3}$
volume:	1.2136e-02	1.5528e-02	1.5528e-02	2.4875e-02

6.2 Visualizations

Optimal solutions: (`script`)

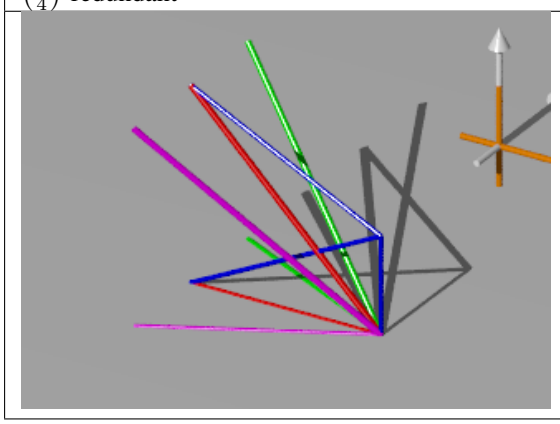


Robust solutions: (`script`)

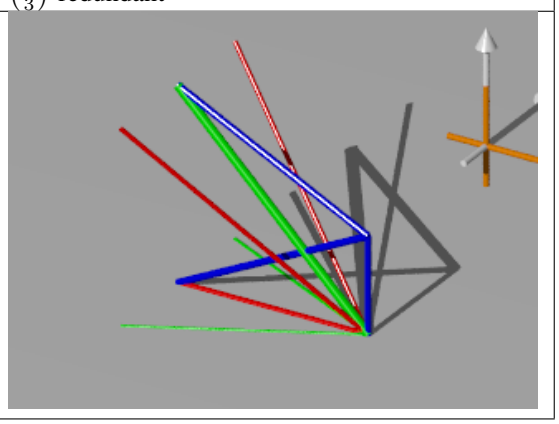


Redundant solutions: (`script`)

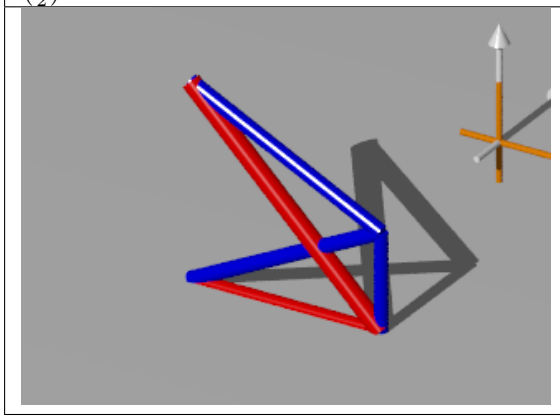
$\left(\frac{1}{4}\right)$-redundant



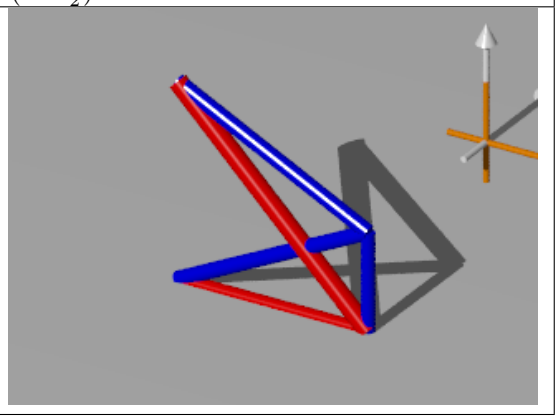
$\left(\frac{1}{3}\right)$-redundant

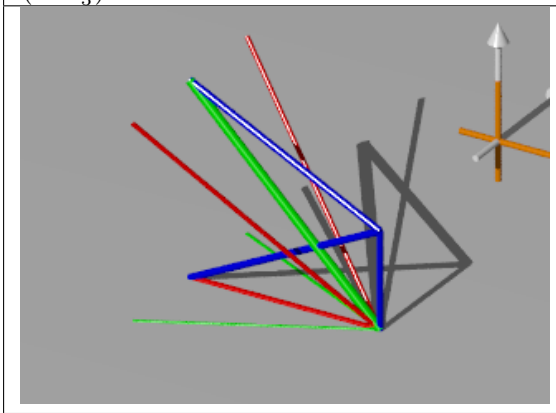
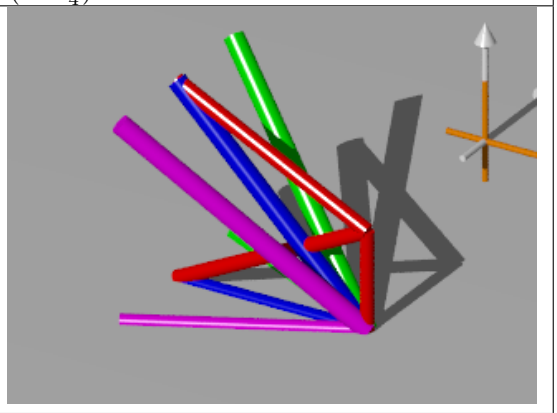


$\left(\frac{1}{2}\right)$-redundant

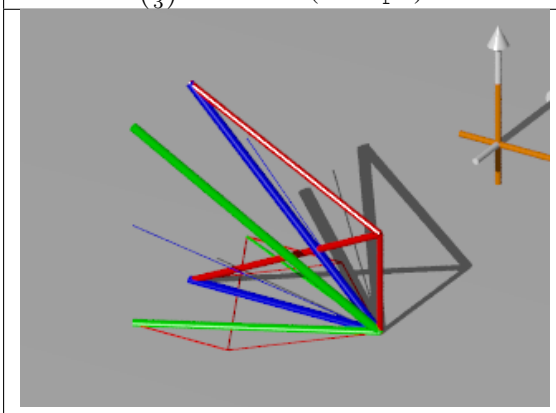
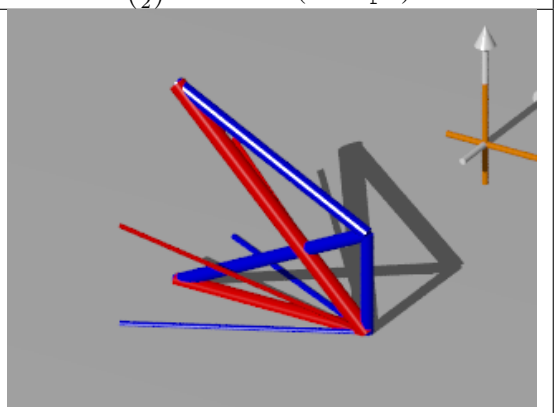
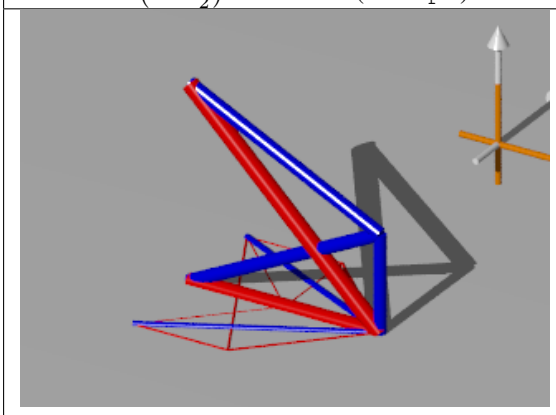
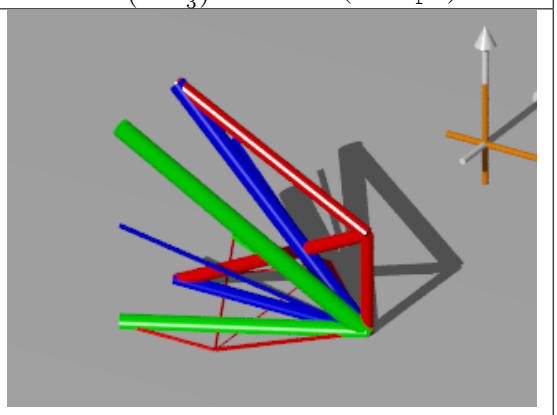


$\left(1-\frac{1}{2}\right)$-redundant



$\left(1-\frac{1}{3}\right)$-redundant	$\left(1-\frac{1}{4}\right)$-redundant
	

Robust and redundant solutions:

robust and $\left(\frac{1}{3}\right)$-redundant: (`script`)	robust and $\left(\frac{1}{2}\right)$-redundant: (`script`)
	
robust and $\left(1-\frac{1}{2}\right)$-redundant: (`script`)	robust and $\left(1-\frac{1}{3}\right)$-redundant: (`script`)
	

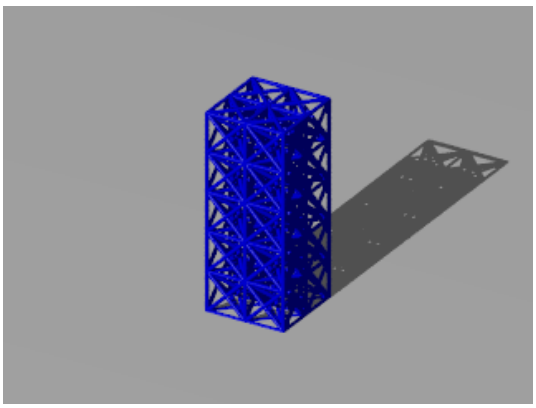
6.3 Python scripts

- `batch_example03.py`
- `batch_example03_robust.py`
- `batch_example03_redundant.py`
- `batch_example03_redundant_1div3_robust.py`
- `batch_example03_redundant_1div2_robust.py`
- `batch_example03_redundant_1minus1div2_robust.py`
- `batch_example03_redundant_1minus1div3_robust.py`

CHAPTER

SEVEN

EXAMPLE OF A MAST



In this example of a mast are 72 nodes $(a, b, c) \in \mathbb{R}^3$, $a, b \in \{0, \frac{1}{7}, \frac{2}{7}\}$, $c \in \{\frac{x}{7} : x = 0, 1, \ldots, 7\}$ measured in meter and 503 potential bars between every two not fixed nodes with a maximal distance of $\frac{\sqrt{3}}{7}$ — long bars which are located alongside several shorter bars are ignored. The supports are in the nodes $(a, b, 0)$, $a, b \in \{0, \frac{2}{7}\}$ which are fixed in every direction. In the node $\left(\frac{1}{7}, \frac{1}{7}, 1\right)$ acts a force of 10^5 N in the negative z-direction.

`The main problem is implemented as a script`. The other scripts are linked by the pictures.

For a perturbation of $50\,\%$ in every direction of the force, and scaling to obtain the same maximal quantities as in the unperturbed case, our robust optimization method produces the robust optimal solution.

7.1 Comparison of the volumes

	optimal	robust
volume:	1.4000e-03	3.4975e-03

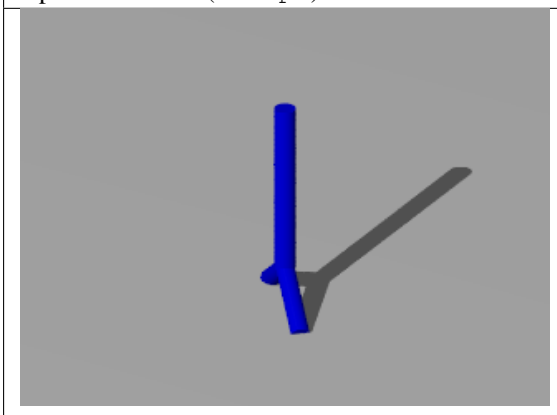
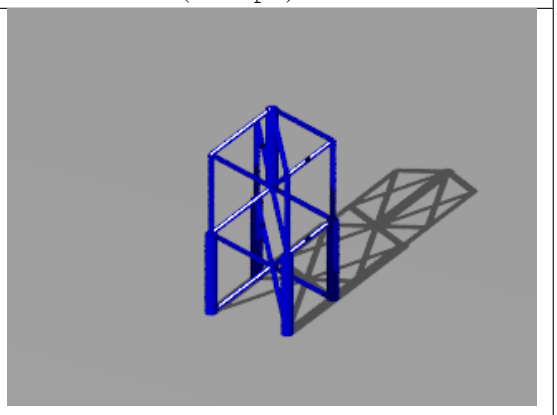
Comparison of the volumes for different redundancies:

redundancy	$\frac{1}{3}$	$\frac{1}{2}$	$1 - \frac{1}{2}$	$1 - \frac{1}{3}$
volume:	2.3000e-03	3.0000e-03	3.0000e-03	4.6000e-03

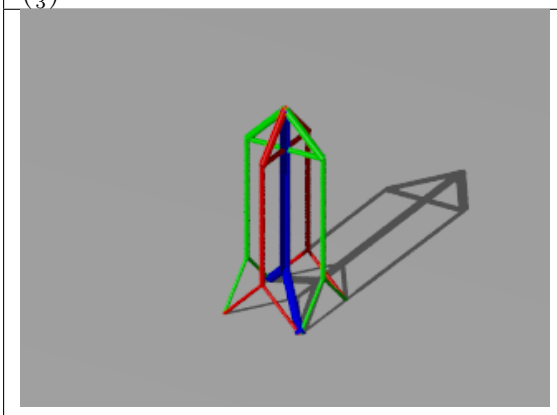
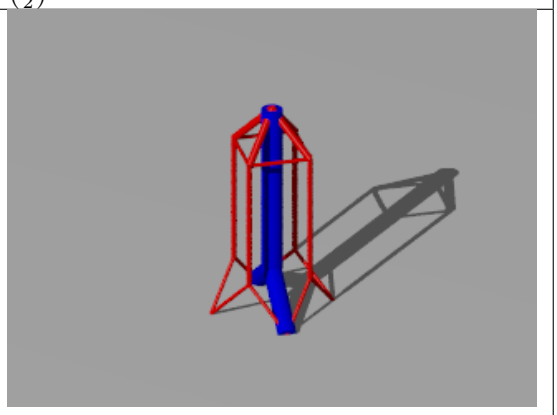
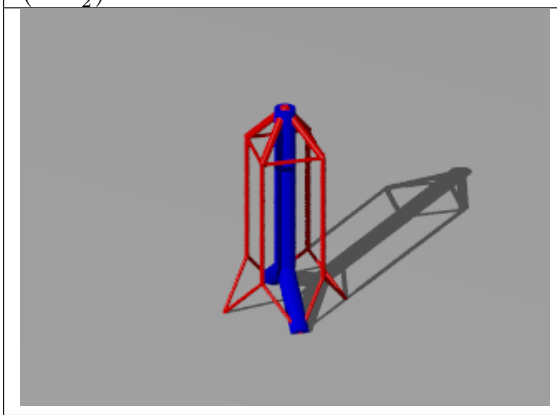
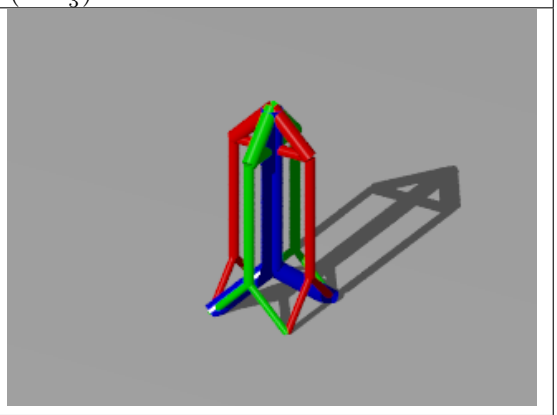
Comparison of the volumes for different redundancies and robustness:

redundancy	$\frac{1}{2}$	$1 - \frac{1}{2}$
volume:	8.0805e-03	8.0805e-03

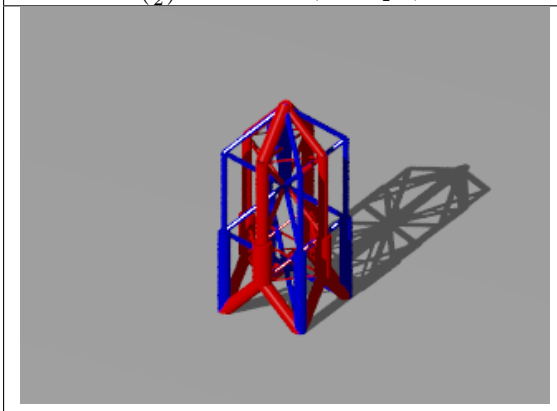
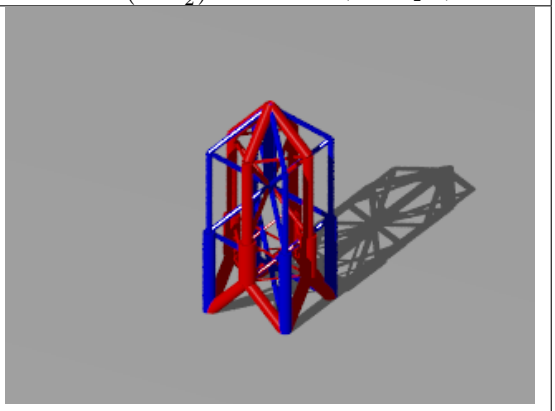
7.2 Visualizations

Optimal solution (`script`)	Robust solution (`script`)
	

Redundant solutions: (`script`)

$\left(\frac{1}{3}\right)$-redundant	$\left(\frac{1}{2}\right)$-redundant
	
$\left(1-\frac{1}{2}\right)$-redundant	$\left(1-\frac{1}{3}\right)$-redundant
	

Robust and redundant solutions:

robust and $\left(\frac{1}{2}\right)$-redundant: (`script`)	robust and $\left(1-\frac{1}{2}\right)$-redundant: (`script`)
	

7.3 Python scripts

- `mast_aufgabenstellung.py`
- `batch_example05.py`
- `batch_example05_robust.py`
- `batch_example05_redundant.py`
- `batch_example05_redundant_1div2_robust.py`
- `batch_example05_redundant_1minus1div2_robust.py`

CHAPTER

EIGHT

MATHEMATICAL BACKGROUND

The software package `ttrr` emerged from the scientific research for [Mohr2011], [MohrSteinMatziesKnapek2012] and [MohrSteinMatziesKnapek2013].

First, I started with MatLab http://www.mathworks.de/ in my PhD research, but the problems were too hard for this great software package. Furthermore, MatLab does not support the solving of mixed integer programs on its own. Therefore I was looking for something else: I found GLPK http://www.gnu.org/software/glpk/ and Python http://www.python.org/ — both are great software!

Because it was simple, I did the visualization with MatLab http://www.mathworks.de/ in my PhD-thesis [Mohr2011]. To write `ttrr`, I do not want to use a commercial back end for the visualization or moreover point you to such a software. But if you like, you can do it on your own.

Contents

8.1 General Concept of Topology Optimization of Truss

Truss topology optimization has its root in [Michell1904].

The analytically stated and known (cf. [Przemieniecki1968]; [Marti2003]; [BendsoeSigmund2004]) linear problem of the topology optimization of a truss with respect to its volume (plastic design) is given by the objective function to be minimized

$$\min_{s,w} l^T s$$

with the bar lengths $l \in \mathbb{R}^n$ subject to the equilibrium condition

$$Cw = f$$

with the reduced geometry matrix $C \in \mathbb{R}^{m \times n}$, the inner bar forces $w \in \mathbb{R}^n$ and the reduced applied external loads $f \in \mathbb{R}^m$. Further given are the constraints

$$\sigma_- s \leq w \leq \sigma_+ s$$

for the linear elasticity by — for all bars equal — stress limits $0 > \sigma_- \in \mathbb{R}$ and $0 < \sigma_+ \in \mathbb{R}$ (e. g. yield points for pressure and tension of the material all bars are made of), and the box constraint

$$0 \leq s \leq s_{\max}$$

for the cross sections of the bars $s \in \mathbb{R}^n$ with the given maximal bar cross section $s_{\max} \in \mathbb{R}^n$. [1]

Here w is seen as a design variable; otherwise we have $s(w)$ as function and therefore no linear program.

If more than one load (e. g. the set F of loads) is to be considered, one ends up at the known multiple load case. But how is it performed? How can we put more than one load in the above linear program? It is impossible! But we can ask for a robust solution — this means we consider the worst case: A solution for all $f^{(i)} \in F$ is searched. Now we really end up at the well known multiple load case: [2]

$$\begin{aligned} \text{obj. func.: } V(s) = l^T s &\to \min_{s,w} \\ \text{s.t.: } \forall f^{(i)} \in F &: \\ C w^{(i)} &= f^{(i)} \in \mathbb{R}^m \\ \sigma_- s \leq w^{(i)} &\leq \sigma_+ s \\ 0 \leq s &\leq s_{\max} \end{aligned}$$

All this is done by `ttrr.ttrr()`.

8.2 Robust Topology Optimization of Truss

For robust optimization see [Ben-TalElGhaouiNemirovski2009].

The implementation in `ttrr.ttrr_robust()` follows [MohrSteinMatziesKnapek2012]. This means if perturbed loads $\tilde{F}$ are given, they are composed by some perturbations of the unperturbed force f. Due to the linear programming we can restrict ourselves to the extreme points of $\tilde{F}$.

If we start the other way round with an additional assumption, it becomes clear: Let f be a given force with a perturbation of 10%. Then we are interested in a solution for all $\tilde{f} \in \{\tilde{f} : 0.9f \leq \tilde{f} \leq 1.1f\}$. Obviously the extreme points are $0.9f$ and $1.1f$. Therefore a solution for all loads in the set $F := \{0.9f; 1.1f\}$ is searched.

So robust optimization in topology optimization is the same as the multiple load case with appropriate load cases. For F a polyhedron a finite number of extreme points is given. And therefore `ttrr.ttrr()` can be recycled.

In addition `ttrr.ttrr_robust()` provides features to calculate the extreme points of simple sets of perturbed forces.

8.3 Redundant Topology Optimization of Truss

The basic concept of topology optimization with regard to redundancy is described in [Mohr2011] and in [MohrSteinMatziesKnapek2013].

Redundant Topology Optimization of Truss means we are looking for the best topology of a truss with a redundancy as to the falling out of some parts of the structure. The dream is a structure which is functional even if a significant damage occurs to an arbitrary set of bars.

You should think of RAID [3]. [PattersonGibsonKatz1987]

For example, a RAID 1 is a bunch of discs and every disc does the work. This means every other one can fail. Following [Mohr2011] this is called "1-1/n"-redundancy (c.f. [Shannon1948] [PattersonGibsonKatz1987]). Transferred to structures this means we are looking for n different structures in the same design space fulfilling the task to sustain the load(s).

[1] The maximal cross section is no limitation — it's a feature! It provides the possibility to limit the cross section; but if you do not want this set a huge upper limit — but an adequate limit helps the optimization algorithms.

[2] Sorry for the compactness, but you can look at the literature all over the world. :-)

[3] RAID is an acronym for Redundant Array of Independent Disks. http://en.wikipedia.org/wiki/RAID

Let $s_i \in \mathbb{R}^N$ be the cross sections of the bars for the structure i. The sum $s = \sum_{i=1}^{n} s_i$ describes the cross sections of the bars for the complete structure. The complete structure is composed of the n structures s_i. With the selecting variables $x_i \in \{0;1\}^N$ we can now formulate the problem as a mixed integer linear program: [4]

$$\begin{aligned}
\text{obj. func.: } V(s_1, s_2, \ldots, s_n) &= \sum_{i=1}^{n} l^T s_i \to \min_{x,w,s} \\
\text{s.t.: } \forall i &= 1, 2, \ldots, n : \\
& s_i \le s_{\max} x_i \in \mathbb{R}^N \\
& \forall k = 1, 2, \ldots, n_l : \\
& \quad C w_i^{(k)} = f^{(k)} \in \mathbb{R}^m \\
& \quad \sigma_- s_i \le w_i^{(k)} \le \sigma_+ s_i \\
& \sum_{i=1}^{n} x_i \le 1 \\
& x \in \{0;1\}^{nN}; \qquad 0 \le s
\end{aligned}$$

From $x \in \{0;1\}^{nN}$ follows $0 \le \sum_{i=1}^{n} x_i$ and $s \le s_{\max}$.

This is done by `ttrr.ttrr_redundant()` for *redundancy* = $-n \le -2$.

A RAID 5 (c.f. [PattersonGibsonKatz1987]) is more complex. It can be described as a simple code. To transfer this concept to structures we only need to know the behavior. A RAID 5 is a bunch of discs. Every single disc may fail and the others do the job together. Following [Mohr2011] this is called "1/n"-redundancy (c.f. [Shannon1948] [PattersonGibsonKatz1987]). Back to structures this means we are looking for n different structures in the same design space. Every single structure can fall out and the other ones should sustain the load(s) together.

Let $s_i \in \mathbb{R}^N$ be the cross sections of the bars for the structure i. As before $s = \sum_{i=1}^{n} s_i$ describes the cross sections of the bars for the complete structure. The complete structure s is composed of the n structures s_i. If a structure i fall out, the resulting structure is $\bar{s}_i = \sum_{j=1}^{n} s_j - s_i$. As mentioned before, this damaged structure should sustain the load(s).

With the selecting variables $x_i \in \{0;1\}^N$ we can now formulate the problem as a mixed integer linear program: [5]

$$\begin{aligned}
\text{obj. func.: } V(s_1, s_2, \ldots, s_n) &= \sum_{i=1}^{n} l^T s_i \to \min_{x,w,s} \\
\text{s.t.: } \forall i &= 1, 2, \ldots, n : \\
& s_i \le s_{\max} x_i \in \mathbb{R}^N \\
& \forall k = 1, 2, \ldots, n_l : \\
& \quad C w_i^{(k)} = f^{(k)} \in \mathbb{R}^m \\
& \quad \sigma_- \bar{s}_i \le w_i^{(k)} \le \sigma_+ \bar{s}_i \\
& \sum_{i=1}^{n} x_i \le 1 \\
& x \in \{0;1\}^{nN}; \qquad 0 \le s \\
\text{with: } \bar{x}_i &= \sum_{j=1, j\ne i}^{n} x_j; \qquad \bar{s}_i = \sum_{j=1, j\ne i}^{n} s_j
\end{aligned}$$

Trivially $0 \le \sum_{i=1}^{n} \bar{x}_i \le 1$ holds and therefore $0 \le \bar{x}_i \le 1$. From $x \in \{0;1\}^{nN}$ follows $0 \le \sum_{i=1}^{n} x_i$ and $s \le s_{\max}$. The damaged structures are also well defined:

$$\bar{s}_i = \sum_{j=1, j\ne i}^{n} s_j \le s_{\max} \sum_{j=1, j\ne i}^{n} x_j = s_{\max} \bar{x}_i \le s_{\max}$$

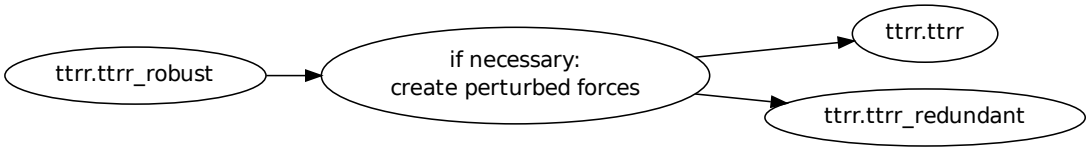
[4] MIP: mixed integer problem; MILP: mixed integer linear program. Because MIP in general is too time consuming everybody is only thinking of MILP and often calls it MIP.

Clearly $0 \leq \bar{s}_i$ holds. So we get for the damaged structures $0 \leq \bar{s}_i \leq s_{\max}$ and for the complete structure $0 \leq s \leq s_{\max}$.

This is done by `ttrr.ttrr_redundant()` for redundancy = $n \geq 2$.

8.4 Robust Redundant Topology Optimization of Truss

Robust redundant topology optimization is nothing more than redundant topology optimization with appropriate load cases. This concept of redundant robust topology optimization is described in [MohrSteinMatziesKnapek2013].



8.5 Uniqueness of the solution

For a linear problem possibly more than one edge of the polyhedron exists as a solution. So the solution of a linear problem is not unique — but the optimal object value is.

Let us consider the small example:

$$\begin{aligned} \text{obj. func.: } z^T x &:= \begin{pmatrix} 1 & 1 \end{pmatrix} x \to \max_x \\ \text{s.t.: } Ax &:= \begin{pmatrix} 1 & 1 \end{pmatrix} x \leq b := 1 \\ x &\geq 0 \end{aligned}$$

A visualization is given in Fig. *Visualization of the simple LP*.

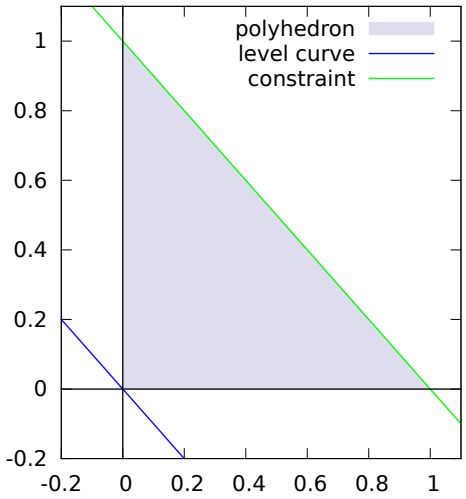
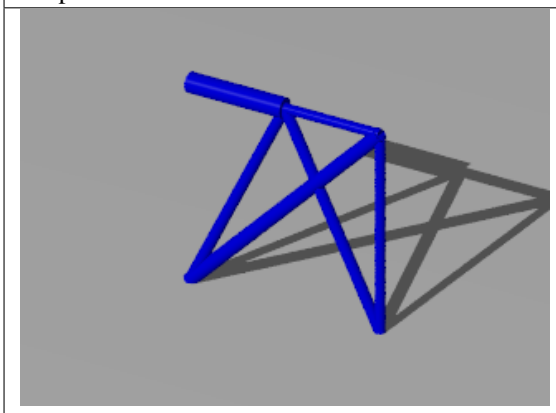
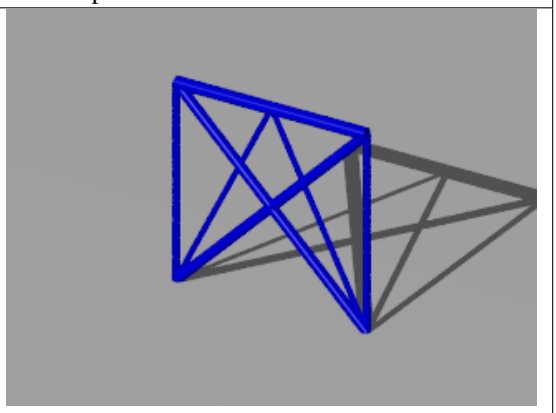


Figure 8.1: Visualization of the simple LP
In the visualization we see a level curve for $z^T x = 0$ and the polyhedron $\{x \in \mathbb{R}^2 : Ax \leq b \wedge x \geq 0\}$ bounded by the constraints.

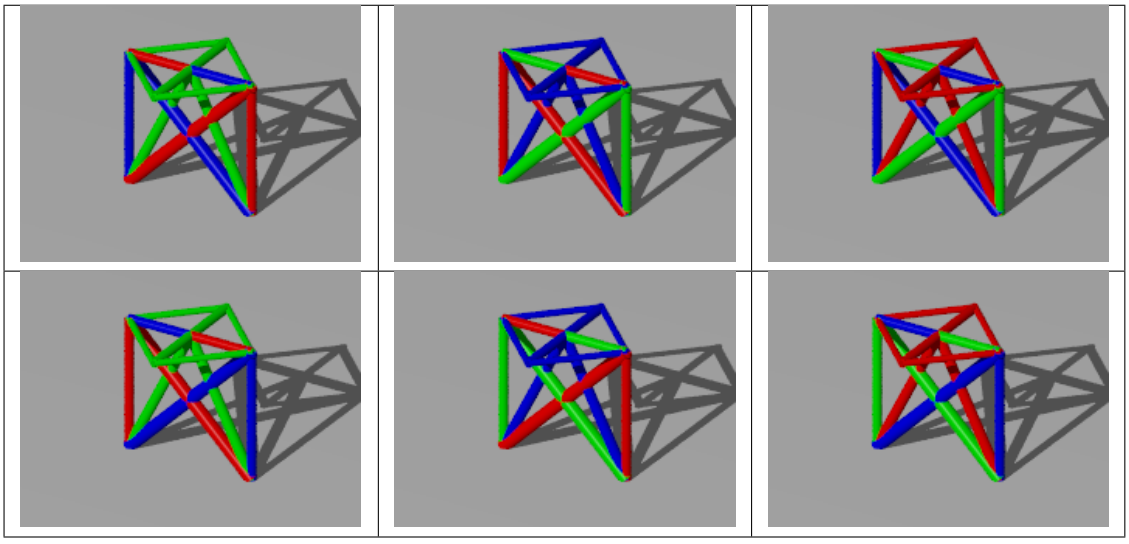
Without a lengthy calculation we can see that the optimal value of the objective function $z^T x$ is 1. The edges $(0, 1)$ and $(1, 0)$ are feasible, optimal and have both the same value of the objective function — the same for all points between these edges.

So in general you cannot expect a unique solution for a linear program. The simplex algorithm will find an edge as a solution. An interior point algorithm will typically find a solution between edges which are optimal. The following two pictures from *Example: 2_load_cases_3_dim* show this behavior — both are optimal for the same problem.

simplex	interior point
	

For the redundant topology optimization we have in every case more than one solution. For example let $\tilde{x}, \tilde{w}, \tilde{s}$ with $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_n)$ be a solution, then $(\tilde{s}_2, \tilde{s}_1, \ldots, \tilde{s}_n)$ is one, too.

For the example *Example: 2_load_cases_3_dim* for a "1/3"-redundancy this is shown in the following table:



To mostly overcome this behavior we can require an order of the volumes of the structures:

$$l^T s_1 \leq l^T s_2 \leq \ldots \leq l^T s_n$$

This is implemented in `ttrr.ttrr_redundant()` with the flag "sort".

8.6 Scaling Factors

To look at scaling behavior of the above linear programs let us consider the following structure:

$$\begin{aligned} \text{obj. func.: } V(s) = l^T s &\to \min_{s,w} \\ Cw &= f \\ \sigma_- s \leq w &\leq \sigma_+ s \\ 0 \leq s &\leq s_{\max} \end{aligned}$$

8.6.1 Material, Load and Design Space

The material parameters for the topology optimization of truss given to `ttrr.ttrr()`, `ttrr.ttrr_robust()` or `ttrr.ttrr_redundant()` are:

- $s_{\max}$: the maximal allowed cross section of a bar
- σ_- : lower limit for the allowed stress, e.g. compressive strength
- σ_+ : upper limit for the allowed stress, e.g. tensile strength
- density : density

The density is only used to calculate the mass of the structure out of its volume.

$s_{\max}$ is necessary for `ttrr.ttrr_redundant()` and gives an additional feature to `ttrr.ttrr()` and `ttrr.ttrr_robust()`. If its not wanted a huge value would disable it. But in any case it gives with $0 \leq s \leq s_{\max}$ bounding values. So we have a closed feasible region.

σ_- and σ_+ are really used material parameters. Typical topology optimization is a preliminary design. Therefore it must not be totally accurate to the praxis. Typically $\sigma_- = -\sigma_+ < 0$ is chosen. So we can assume $\sigma_- = -\sigma_+ = -\sigma$ for some $\sigma > 0$.

$$\begin{aligned} \text{obj. func.: } V(s) = l^T s &\to \min_{s,w} \\ Cw &= f \\ -\sigma s \leq w &\leq \sigma s \\ 0 \leq s &\leq s_{\max} \end{aligned}$$

Easily to see the following program results in an equivalent optimum. With $\tilde{s} = \sigma s$ we get:

$$\begin{aligned} \text{obj. func.: } V(\tilde{s}) = l^T \tilde{s} &\to \min_{\tilde{s},w} \\ Cw &= f \\ -\tilde{s} \leq w &\leq \tilde{s} \\ 0 \leq \tilde{s} &\leq \frac{1}{\sigma} s_{\max} \end{aligned}$$

This is only a scaling. The magnitude of the loads is only a scaling, too. With $\tilde{w} = \alpha w$ for some $\alpha > 0$ we get:

$$\begin{aligned} \text{obj. func.: } V(\tilde{s}) = l^T \tilde{s} &\to \min_{\tilde{s},\tilde{w}} \\ C\tilde{w} &= \alpha f \\ -\alpha\tilde{s} \leq \tilde{w} &\leq \alpha\tilde{s} \\ 0 \leq \tilde{s} &\leq \frac{1}{\sigma} s_{\max} \end{aligned}$$

And furthermore with $\hat{s} = \alpha\tilde{s} = \alpha\sigma s$ the following program is equivalent, too:

$$\begin{aligned} \text{obj. func.: } V(\hat{s}) = l^T\hat{s} &\to \min_{\hat{s},\tilde{w}} \\ C\tilde{w} &= \alpha f \\ -\hat{s} \le \tilde{w} &\le \alpha\hat{s} \\ 0 \le \hat{s} &\le \frac{1}{\alpha\sigma}s_{\max} \end{aligned}$$

Keep in mind it is not required to change the objective function. For $\beta > 0$ the minimum of the functions $g(x)$ and $\beta g(x)$ is reached at the same point.

The size of the design space is not found directly in the linear program. In the geometry matrix C occur only the angles of the bars. So the magnitude of the size of the design space in meter or millimeter will result in the same optimal structure, because l is only in the objective function and will be changed by a scalar factor.

8.6.2 Numeric

A scaling of

$$\begin{aligned} \text{obj. func.: } V(s) = l^T s &\to \min_{s,w} \\ Cw &= f \\ \sigma_- s \le w &\le \sigma_+ s \\ 0 \le s &\le s_{\max} \end{aligned}$$

do not change the (relative) condition of the problem. But it can change the absolute condition and therefore helps the computer by the calculation.

To do so the functions `ttrr.ttrr()`, `ttrr.ttrr_robust()` or `ttrr.ttrr_redundant()` have the parameters $\alpha =$ scale_sigma and $\beta =$ scale_forces. To get the picture let us see how they transform the above problem:

$$\begin{aligned} \text{obj. func.: } V(s) = l^T s &\to \min_{s,w} \\ C\beta w &= \beta f \\ \alpha\sigma_- \frac{\beta}{\alpha}s \le \beta w &\le \alpha\sigma_+ \frac{\beta}{\alpha}s \\ 0 \le s &\le s_{\max} \end{aligned}$$

Now the calculation will be done with $\tilde{s} = \frac{\beta}{\alpha}s$, $\tilde{w} = \beta w$, $\tilde{f} = \beta f$, $\tilde{\sigma}_- = \alpha\sigma_-$, $\tilde{\sigma}_+ = \alpha\sigma_+$ and $\tilde{s}_{\max} = \frac{\beta}{\alpha}s_{\max}$:

$$\begin{aligned} \text{obj. func.: } V(\tilde{s}) = l^T\tilde{s} &\to \min_{\tilde{s},\tilde{w}} \\ C\tilde{w} &= \tilde{f} \\ \tilde{\sigma}_-\tilde{s} \le \tilde{w} &\le \tilde{\sigma}_+\tilde{s} \\ 0 \le \tilde{s} &\le \tilde{s}_{\max} \end{aligned}$$

Afterwards $\tilde{s}$ and $\tilde{w}$ will be transformed back.

For a maximum magnitude of 1 of an element from $\tilde{f}$ the absolute condition of the equation will be a little bit better. For a maximum magnitude of 1 of an element from $\tilde{\sigma}_-$ and from $\tilde{\sigma}_+$ the condition of the inequation will be significant better. If $\alpha = \beta$ the design variables s and $s_{\max}$ resp. will not be changed.

CHAPTER

NINE

README: TTRR - TRUSSTOPOLOGY-ROBUSTREDUNDANT

ttrr is a software package to do topology optimization of a truss with respect to robustness and redundancy.

9.1 install

see *INSTALL: ttrr - TrussTopologyRobustRedundant*

9.2 literature

- Mohr, Daniel P. ; Stein, Ina ; Matzies, Thomas ; Knapek, Christina A.: Redundant robust topology optimization of truss. In: Optimization and Engineering (2013), 1-28. http://dx.doi.org/10.1007/s11081-013-9241-7
- Mohr, Daniel P. ; Stein, Ina ; Matzies, Thomas ; Knapek, Christina A.: Robust Topology Optimization of Truss with regard to Volume. In: arXiv - Mathematics, Optimization and Control (2012). http://arxiv.org/abs/1109.3782v2
- Mohr, Daniel P.: Redundante Topologieoptimierung. Neubiberg, Universitaet der Bundeswehr Muenchen, Fakultaet fuer Luft- und Raumfahrttechnik, Diss., Dezember 2011. http://nbn-resolving.de/urn/resolver.pl?urn=urn:nbn:de:bvb:706-2664

9.3 copyright + license

Author: Daniel Mohr.

Date: 2013-11-09 (last change).

License: GNU GENERAL PUBLIC LICENSE, Version 3, 29 June 2007.

CHAPTER

TEN

INSTALL: TTRR - TRUSSTOPOLOGY-ROBUSTREDUNDANT

10.1 info

ttrr is a software package to do topology optimization of a truss with respect to robustness and redundancy.

10.2 before you install

Before you install, make sure to have:

- Python http://www.python.org/
- NumPy http://numpy.scipy.org/ http://pypi.python.org/pypi/numpy
- SciPy http://www.scipy.org/ http://pypi.python.org/pypi/scipy
  - and also for NumPy/SciPy: BLAS and LAPACK libraries
- GLPK http://www.gnu.org/software/glpk/

To get graphical output (picture, movie, animation) you need some of the following programs:

- gnuplot http://www.gnuplot.info/
- POV-Ray http://www.povray.org/
- png2yuv from MJPEG http://mjpeg.sourceforge.net/
- ffmpeg2theora http://www.v2v.cc/~j/ffmpeg2theora/
- png2theora from libtheora http://www.theora.org
- FFmpeg http://ffmpeg.org/
- APNG Assembler http://sourceforge.net/projects/apngasm/
- convert from ImageMagick http://www.imagemagick.org/

At least for pictures you need gnuplot and/or POV-Ray. The frames for all movie and animation output are rendered with POV-Ray.

There is also a small extra command to check for availability of necessary python modules:

```
python setup.py check_modules
```

If you want to use this complete software you should have no modules which are not available.

10.2.1 Python3

If you use Python3, you must convert to Python3 via 2to3:

```
tar xzf ttrr-*.tar.*
cd ttrr-*/
2to3 -w .
```

10.3 install

10.3.1 global-install

To install this software global to / the following steps are to perform:

```
tar xzf ttrr-*.tar.*
cd ttrr-*/
python setup.py build_c_scripts
python setup.py install
```

Please recognize the extra step *python setup.py build_c_scripts* here.

10.3.2 home-install

To install this software to your $HOME the following steps are to perform:

```
tar xzf ttrr-*.tar.*
cd ttrr-*/
python setup.py build_c_scripts
python setup.py install --home=~
```

Please recognize the extra step *python setup.py build_c_scripts* here.

10.4 hints

- GLPK 4.38 and 4.40 do not work.

  GLPK 4.43 and 4.44 and 4.45 and 4.47 work.

  GLPK 4.52.1 gives other binary results compared to "results.tar.bz2". But these are only numerical rounding errors.

- Keep in mind to have the right pathes.

  For the above installation to $HOME the software installs in:

  ```
  ~/bin
  ~/lib/python
  ```

  Please make sure to have these pathes in $PATH and $PYTHONPATH, respectively. For example:

  ```
  export PATH=$PATH:~/bin
  export PYTHONPATH=~/lib/python
  ```

- You can use your own compiler-flags. For example:

  ```
  CFLAGS="-march=amdfam10 -O3 -msse4a -mfpmath=sse -pipe -fomit-frame-pointer" python setup.py
  ```

- If you have libraries in a not default path, please let it know.

  For example (e. g. OpenBSD or FreeBSD):

  ```
  export LIBRARY_PATH=$LIBRARY_PATH:/usr/local/lib
  export CPATH=$CPATH:/usr/local/include
  ```

- Because we are doing scientific computing (repeatable: conform to IEEE-754 standard; with the same result each time the computing is done — independent of the load of the machine) the default floating point arithmetic unit is set to sse by the use flag "-mfpmath=sse" in "setup.py".

  If you really wish to use 387 read the gcc man page to understand what you do. In principle if you calculate with 387 the results may be different every time! Caused by the simplex algorithm mostly used in ttrr through GLPK the basis of the result will mostly be the same. The final calculation of the objective function value is not very time consuming. Therefore the results could be the same.

- SciPy has missing dependencies in Gentoo Linux amd64 at the moment (2012-02-26) (see https://bugs.gentoo.org/show_bug.cgi?id=380597). You need sci-libs/clapack and sci-libs/lapack-atlas, too. For example:

  ```
  emerge sci-libs/clapack sci-libs/lapack-atlas
  eselect blas set atlas
  eselect cblas set atlas
  eselect lapack set atlas
  emerge sci-libs/scipy
  ```

- Fedora needs not only glpk but glpk-devel and gcc, too. gcc should come as a dependency of SciPy, but it does not.

- Fedora lacks multimedia software. The repository http://rpmfusion.org/ provides MJPEG, ffmpeg2theora and FFmpeg (and more), but not povray. It is possible to install it manually (see other hint).

- Since in debian the module argparse is not included in python, one needs the package python-argparse in addition.

- However, in debian png2yuv from MJPEG http://mjpeg.sourceforge.net/ is not in the package management system. But it can be installed as mjpegtools from: http://deb-multimedia.org

- If povray is not in the package management system (ubuntu 12.04, Fedora 16, Fedora 17, LMDE 201204, debian 6.0.5), it can be installed from http://www.povray.org/ http://www.povray.org/redirect/www.povray.org/ftp/pub/povray/Official/Linux/povlinux-3.6.tgz

  Unfortunately the install script does not work with a simple shell. Replacing sh with bash in the first line of the install script gives a working script in many distributions. (Sometimes /bin/sh is a link to /bin/bash and then it is working out of the box.)

- openSUSE 12.1 x86_64 behave in a different way from typical linux.

  The software SciPy and GLPK are not in the package management system; but are available from the repository http://download.opensuse.org/repositories/science/ (see http://www.scipy.org/Installing_SciPy/Linux#head-3dbbf9abe395e974d3ec911ae56863f4af70df9b).

  Not only glpk but also glpk-devel is needed.

  To install ttrr software to your $HOME in openSUSE the following steps are to perform:

  ```
  tar xzf ttrr-*.tar.*
  cd ttrr-*/
  python setup.py build_c_scripts
  python setup.py install --prefix=~
  ```

  Please make sure to have these pathes in $PATH and $PYTHONPATH, respectively. For openSUSE you need:

  ```
  export PATH=$PATH:~/bin
  export PYTHONPATH=~/lib/python2.7/site.packages
  ```

The software ffmpeg2theora and FFmpeg are in the repository: http://packman.links2linux.org/

10.5 tested and developed

Tested and developed with Gentoo Linux http://www.gentoo.org/ and the gentoo-packages

- dev-lang/python
- dev-python/numpy
- sci-libs/scipy
- sci-mathematics/glpk
- sci-visualization/gnuplot
- media-gfx/povray
- media-video/mjpegtools
- media-video/ffmpeg2theora
- media-video/ffmpeg
- media-libs/libtheora
- media-gfx/apngasm
- media-gfx/imagemagick

in the following versions/USE-flags:

- sci-mathematics/glpk-4.45 USE="doc examples gmp -mysql -odbc -static-libs"
- dev-lang/python-2.7.2-r3 USE="doc examples gdbm ipv6 ncurses readline sqlite ssl threads tk (wide-unicode) xml -berkdb -build -wininst"
- sci-visualization/gnuplot-4.4.4-r1 USE="X cairo doc emacs examples gd ggi latex lua plotutils readline thin-splines wxwidgets xemacs (-svga)"
- media-gfx/povray-3.7.0_rc3 USE="X openexr tiff -debug"
- media-video/mjpegtools-2.0.0-r1 USE="gtk mmx png -dga -dv -quicktime -sdl -sdlgfx -static-libs -v4l"
- media-video/ffmpeg2theora-0.28 USE="-debug -kate"
- dev-python/numpy-1.6.0 USE="doc lapack -test"
- sci-libs/scipy-0.9.0-r1 USE="doc -umfpack"
- media-libs/libtheora-1.1.1 USE="doc encode examples -static-libs"
- media-gfx/apngasm-2.5
- media-video/ffmpeg-0.7.8 USE="3dnow 3dnowext X aac alsa bzip2 doc encode hardcoded-tables jpeg2k mmx mmxext mp3 oss qt-faststart speex ssse3 theora threads truetype vorbis zlib (-altivec) -amr -avx -bindist (-celt) -cpudetection -custom-cflags -debug -dirac -faac -frei0r -gsm -ieee1394 -jack -network -pic -rtmp -schroedinger -sdl -static-libs -test -v4l -vaapi -vdpau -vpx -x264 -xvid" VIDEO_CARDS="-nvidia"
- media-gfx/imagemagick-6.7.5.3 USE="X bzip2 cxx djvu fontconfig graphviz gs jpeg jpeg2k lzma openexr openmp pango perl png raw svg tiff truetype wmf xml zlib -autotrace -corefonts -fftw -fpx -hdri -jbig -lcms -lqr -opencl -q32 (-q64) -q8 -static-libs -test -webp"

10.5.1 tested with

ttrr was successfully tested with:

- Gentoo Linux amd64 http://www.gentoo.org/ (with Python2 and Python3; see hints above)

- ubuntu 11.10 desktop i386 http://www.ubuntu.com/
- ubuntu 11.10 desktop amd64 http://www.ubuntu.com/
- ubuntu 12.04 desktop amd64 http://www.ubuntu.com/ (since povray is not in the package management system, it was manually installed; see hints above)
- ubuntu 13.04 desktop amd64 http://www.ubuntu.com/ (since povray is not in the package management system, it was manually installed)
- FreeBSD 9.0 i386 http://www.freebsd.org/ (see hints above)
- FreeBSD 9.0 amd64 http://www.freebsd.org/ (see hints above)
- OpenBSD 5.0 i386 http://www.openbsd.org/ (see hints above)
- OpenBSD 5.0 amd64 http://www.openbsd.org/ (see hints above)
- OpenBSD 5.1 amd64 http://www.openbsd.org/ (see hints above)
- Fedora 16 x86_64 Desktop http://fedoraproject.org/ (without povray; see hints above)
- Fedora 17 x86_64 Desktop http://fedoraproject.org/ (since povray is not in the package management system, it was manually installed; see hints above)
- LMDE 201204 with MATE 64-bit http://www.linuxmint.com/ (since povray is not in the package management system, it was manually installed; see hints above)
- debian 6.0.5 amd64 http://www.debian.org/ (since povray is not in the package management system, it was manually installed; see hints above)
- Arch Linux 2011.08.19 core x86_64 http://www.archlinux.org/ (with Python3)
- openSUSE 12.1 x86_64 http://www.opensuse.org/ (with required additional repositories; see hints above)

10.6 after install

You can check your installation with the examples in the examples directory:

```
./test
```

The results are compared with the reference results in "results.tar.bz2". You should view the images and pictures with your preferred software, too.

Under GLPK 4.52.1 you get different binary results compared to "results.tar.bz2". But these are only numerical rounding errors.

The most time (about 98 % cpu time) is spent on rendering by povray.

If the bash script "test" is not working for you, you can try single examples, e. g.:

```
python batch_2_load_cases_3_dim.py
python compareresults.py --compare_results -f *.bz2
ttrr_tools.py --png_gnuplot --png_pov --ogv --apng --mng --gif -f 2_load_cases_3_dim_tar.bz2 -xm
```

On a laptop (lenovo T410s, 4 intel-cores with 2.53 GHz, 4 GB RAM) it takes about 4 seconds for test_short and about 178 minutes (real time, 478 minutes user time) for test_long with povray V3.6.

On a desktop computer (8 amd-cores with 3.60 GHz, 16 GB RAM) it takes about 12 seconds for test_short and about 70 minutes (real time, 365 minutes user time) for test_long with povray V3.7.

CHAPTER

ELEVEN

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Author: Daniel Mohr.

Date: 2013-11-09 (last change).

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CHAPTER

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CHAPTER

THIRTEEN

INDICES AND TABLES

- *genindex*
- *modindex*
- *search*

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PYTHON MODULE INDEX

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